Algebraic Thinking

in the

Common Core State Standards

for

Mathematics

Helping You See Algebra Clearly

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Underlying Principle

- "*Everyone is good at mathematics because everyone can think. And mathematics is about thinking.*"
  - Yeap Ban Har, National Institute of Education, Singapore.
- Corollary 1: Strategies that attempt to remove thinking from learning are bound to fail in the long run.
- Corollary 2: When learning is effective, "getting the right answer" is but a small piece of the work.

Major Themes

- All students means ALL students
- The work is about improving instruction, which requires that teachers collaborate to reach more students more of the time

Fundamentals of the Common Core State Standards

CCSS Principles

- **Focus**: focus strongly on key ideas, understandings, and skills in each grade and course
- **Coherence**: think across grades, and link to major topics in each grade
- **Rigor**: in major topics, pursue with equal intensity
  - conceptual understanding,
  - procedural skill and fluency, and
  - applications

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Let’s Do Some Mathematics

A Mathematics Problem

$$4 + 5 = \square + 3$$

- What number goes in the box?
- What other answer are students likely to give?
- At what grade are students able to solve this problem?
- By what grade should students' solutions be easy and automatic?

The Case of Kevin

- Kevin is in Kindergarten
- As you watch the video, consider the following questions:
  - What knowledge and skills support Kevin in this problem?
  - What can you infer about the instruction Kevin has experienced?

- Watch video

Note: Videos are from *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*, by Carpenter, Franke, and Levi, 2003

Kevin’s Solution

- What knowledge and skills led to Kevin’s success?
- What can we infer about the instruction Kevin has experienced?
- With appropriate instruction, can all children do what Kevin did?
- Understanding of the problem situation:
  - “Is the same as”
  - “Hmm”
  - “Number sentence”
- Proficiency with counting
- Tendency to use manipulatives (including fingers)

The Division Problem

- The question was, “What is 42 ÷ 7?”
- Here is a second grader’s solution. He did not write anything. This is what he said:

I don’t know how many 7s are in 42, but I do know:

$40 \div 10 = 4$

$4 \times 3 = 12$

$12 + 2 = 14$

$14 \div 7 = 2$

$2 + 4 = 6$

The answer is 6.

Watch related [video]()

$$40 = 4\times 10$$
$$= 4\times(7 + 3)$$
$$= 4\times 7 + 4\times 3$$

So 4 7s is less than 40 by 4×3, or 12

So 4 7s is less than 42 by 14

… which is 2 more 7s

The Chicken and Dog Problem

- Chickens and dogs are running around outdoors. Together they have 40 heads and 96 feet. How many dogs are there?
- Here is the second grader's solution:

$$x + y = 40$$
$$2x + 4y = 96$$

Imagine all the chickens are standing on one leg and all the dogs are standing on their hind legs.

Now there are 96 ÷ 2, or 48 feet on the ground.

$$x + 2y = 48$$
$$x + y = 40$$

There are 8 extra legs.

The chickens are all on one leg.

The extra legs belong to dogs.

There are 8 dogs.

$$y = 8$$

The Square and Triangle Problem

□ + □ + □ + △ = 47

□ – △ = 1

- From the second equation, I know that square is one more than triangle.
- Then I imagined that the triangle in the first equation was a square.
- Then the sum would be one more, so the 4 squares would equal 48.
- The square is 12 and the triangle is 11.

□ = △ + 1

Add the two equations …

Divide by 4 …

Odds and Evens

- Explain the rule odd + odd = even
  - Hints: Use pictures or real-world contexts
  - Does your explanation show why it will *always* work, for any two odd numbers?
- Watch video

Implications for Practice

- Instruction should take advantage of what students *are* thinking
  - So that symbol manipulations more often represent ideas in their heads
  - Algebra is about thinking and reasoning in general ways
- Instruction should emphasize
  - Standards for Mathematical Practice (CCSS)
  - Mathematical Process Standards (NCTM)
  - Strands of Mathematical Proficiency (NRC)

About Algebra

- Algebra is about structure
  - e.g., properties of operations
- Algebra is about relationships
- Algebra is about thinking and reasoning generally
- Algebra is about patterns
  - Patterns based in contexts
  - Not just guessing what comes next
- Formal algebra begins in grade 8 for all students
  - High school Algebra 1 needs to be different
- Algebraic thinking needs to be present every year, K-12, including in Geometry

Types of Tasks

- Memorization
- Procedures without connections to concepts or meaning
- Procedures with connections to concepts and meaning
- Doing mathematics

  - See Smith and Stein, 2012, *Selecting and Creating Mathematical Tasks: From Research to Practice.*

Reminder

- "These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep."
  (CCSS, 2010, p. 5)

Assessment Considerations

Assessment Design

- Smarter Balanced Assessment Consortium (SBAC)
- Computer adaptive summative and formative assessments
  - Grade-level assessments in grades 3-8
  - High school assessment at grade 11
- **Innovative item types** go beyond multiple choice questions to include constructed response and performance tasks that measure critical thinking and problem solving.
- **Interim assessments** provide information about student progress throughout the year to help teachers differentiate instruction.
- A **digital library** of research-based formative assessment practices and tools.

SBAC Mathematics Claims

- Claim #1: **Concepts and Procedures (40%)**
  - Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency
- Claim #2: **Problem Solving (20%)**
  - Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies
- Claim #3: **Communicating Reasoning (20%)**
  - Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others
- Claim #4: **Modeling and Data Analysis (20%)**
  - Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems

Implementation Suggestions

Tips for Implementation

1. Get to know the CCSS
   - Use the critical areas of focus
   - Take a progressions view
2. Lead with the mathematical practices
   - With the content you are teaching now
3. Work collectively
   - You do not need to invent it all yourself
4. Involve administrators and parents
5. Take some transitional steps
   - Changes you can make soon

Tips for Implementation

6. Build support structures for students who are behind
7. Design programs for *all students,* driven by progressions, not course names
8. Require focus and coherence in district initiatives and professional development offerings
9. Document your implementation
   - Treat your implementation work as action research
10. Take a deep breath … and prepare for a long haul
    - Improving instruction and building new systems takes time