

Learning Progressions for the Common Core State Standards

Bradford Findell

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Brad.Findell@ode.state.oh.us

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Grade Level Overview

Cross-cutting themes

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Critical Area of Focus

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000



Format of K-8 Standards

Grade Level

Operations and Algebraic Thinking

1.OA

Domain

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.²
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Standard

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.³ *Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)*
4. Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.

Cluster



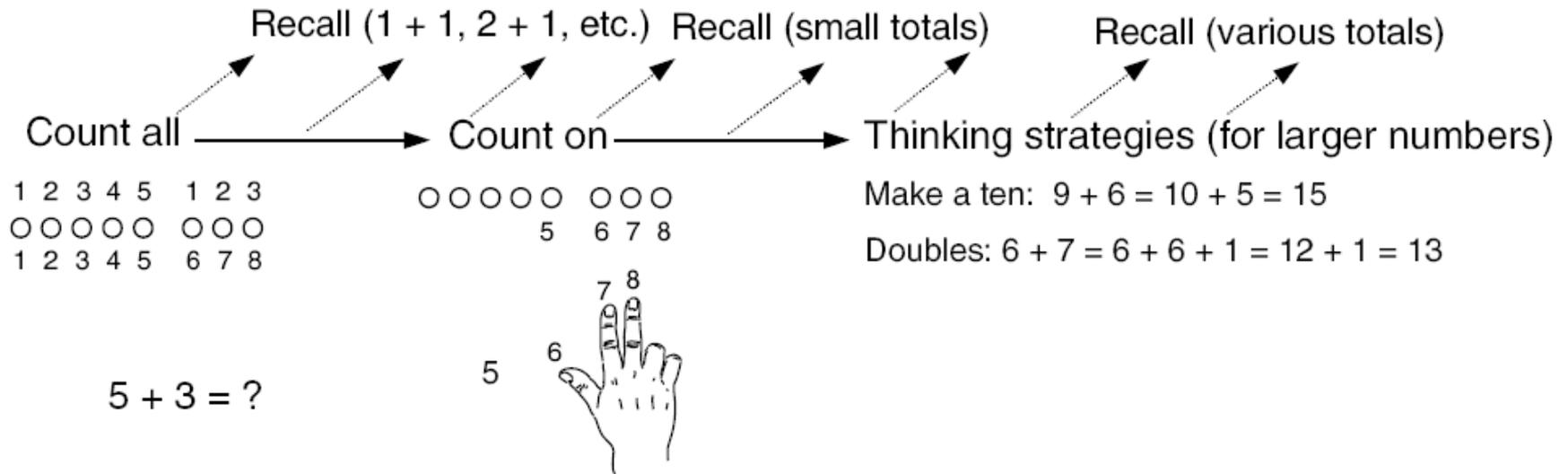
CCSS Domain Progression

K	1	2	3	4	5	6	7	8	HS
Counting & Cardinality									
Number and Operations in Base Ten						Ratios and Proportional Relationships		Number & Quantity	
			Number and Operations – Fractions			The Number System			
Operations and Algebraic Thinking						Expressions and Equations		Algebra	
								Functions	Functions
Geometry									Geometry
Measurement and Data						Statistics and Probability			Statistics & Probability

Progressions

- Progressions
 - Describe a sequence of increasing sophistication in understanding and skill within an area of study
- Three types of progressions
 - Learning progressions
 - Standards progressions
 - Task progressions

Learning Progression for Single-Digit Addition



From *Adding It Up: Helping Children Learn Mathematics*, NRC, 2001.



Standards Progressions



Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details		
1. Ask and answer questions to demonstrate understanding of a text, referring explicitly to the text as the basis for the answers.	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	1. Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.
2. Recount stories, including fables, folktales, and myths from diverse cultures; determine the central message, lesson, or moral and explain how it is conveyed through key details in the text.	2. Determine a theme of a story, drama, or poem from details in the text; summarize the text.	2. Determine a theme of a story, drama, or poem from details in the text, including how characters in a story or drama respond to challenges or how the speaker in a poem reflects upon a topic; summarize the text.
3. Describe characters in a story (e.g., their traits, motivations, or feelings) and explain how their actions contribute to the sequence of events.	3. Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text (e.g., a character's thoughts, words, or actions).	3. Compare and contrast two or more characters, settings, or events in a story or drama, drawing on specific details in the text (e.g., how characters interact).
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, distinguishing literal from nonliteral language.	4. Determine the meaning of words and phrases as they are used in a text, including those that allude to significant characters found in mythology (e.g., Herculean).	4. Determine the meaning of words and phrases as they are used in a text, including figurative language such as metaphors and similes.
5. Refer to parts of stories, dramas, and poems when writing or speaking about a text, using terms such as chapter, scene, and stanza; describe how each successive part builds on earlier sections.	5. Explain major differences between poems, drama, and prose, and refer to the structural elements of poems (e.g., verse, rhythm, meter) and drama (e.g., casts of characters, settings, descriptions, dialogue, stage directions) when writing or speaking about a text.	5. Explain how a series of chapters, scenes, or stanzas fits together to provide the overall structure of a particular story, drama, or poem.
6. Distinguish their own point of view from that of the narrator or those of the characters.	6. Compare and contrast the point of view from which different stories are narrated, including the difference between first- and third-person narrations.	6. Describe how a narrator's or speaker's point of view influences how events are described.
Integration of Knowledge and Ideas		
7. Explain how specific aspects of a text's illustrations contribute to what is conveyed by the words in a story (e.g., create mood, emphasize aspects of a character or setting).	7. Make connections between the text of a story or drama and a visual or oral presentation of the text, identifying where each version reflects specific descriptions and directions in the text.	7. Analyze how visual and multimedia elements contribute to the meaning, tone, or beauty of a text (e.g., graphic novel, multimedia presentation of fiction, folktale, myth, poem).
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast the themes, settings, and plots of stories written by the same author about the same or similar characters (e.g., in books from a series).	9. Compare and contrast the treatment of similar themes and topics (e.g., opposition of good and evil) and patterns of events (e.g., the quest) in stories, myths, and traditional literature from different cultures.	9. Compare and contrast stories in the same genre (e.g., mysteries and adventure stories) on their approaches to similar themes and topics.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, at the high end of the grades 2-3 text complexity band independently and proficiently.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, in the grades 4-5 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, at the high end of the grades 4-5 text complexity band independently and proficiently.

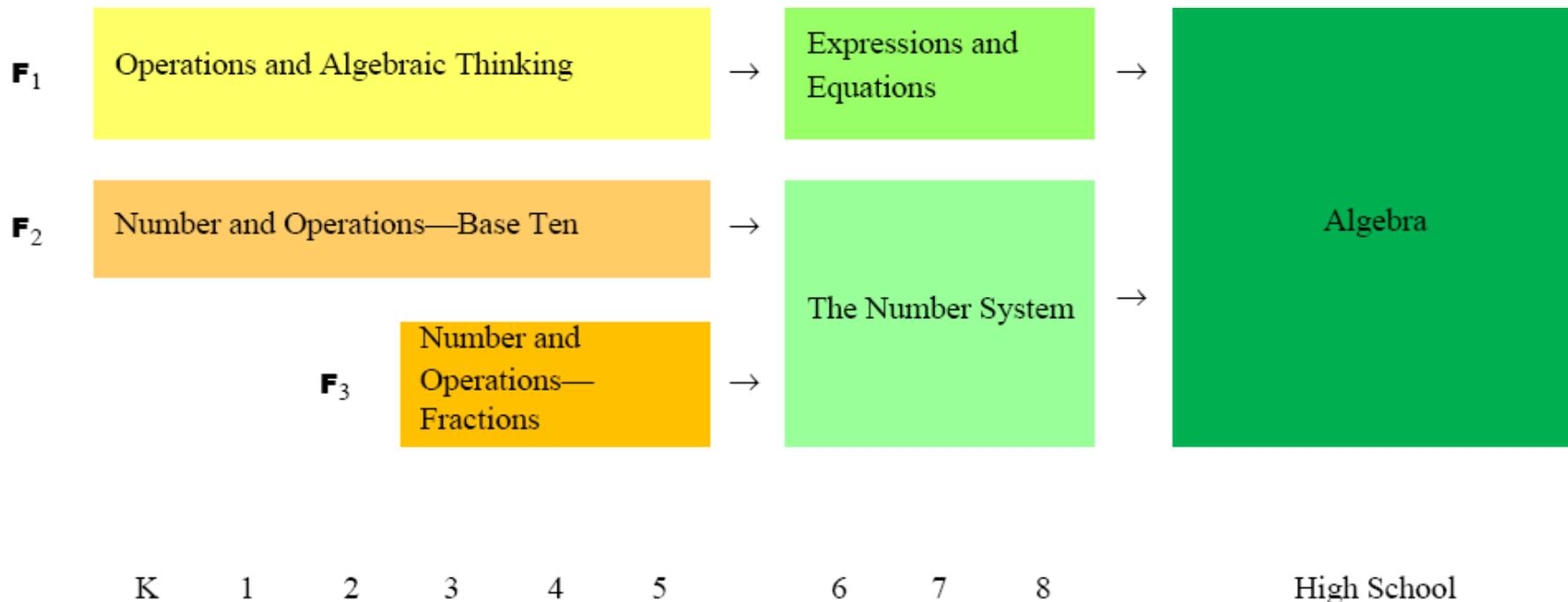
Reading Standards for Literature: Key Ideas and Details

Grade 3	Grade 4	Grade 5
<p>3. Describe characters in a story (e.g., their traits, motivations, or feelings) and explain how their actions contribute to the sequence of events.</p>	<p>3. Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text (e.g., a character's thoughts, words, or actions).</p>	<p>3. Compare and contrast two or more characters, settings, or events in a story or drama, drawing on specific details in the text (e.g., how characters interact).</p>

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Flows Leading to Algebra



Standards Progression: Number and Operations in Base Ten

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade 5
<p>Work with numbers 11-19 to gain foundations for place value.</p> <p>1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p>	<p>Extend the counting sequence.</p> <p>1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.</p> <p>Understand place value.</p> <p>2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:</p> <p>a. 10 can be thought of as a bundle of ten ones — called a “ten.”</p> <p>b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.</p> <p>c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p> <p>3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.</p>	<p>Understand place value.</p> <p>1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 708 equals 7 hundreds, 0 tens, and 8 ones. Understand the following as special cases:</p> <p>a. 100 can be thought of as a bundle of ten tens — called a “hundred.”</p> <p>b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p> <p>2. Count within 1000; skip-count by 5s, 10s, and 100s.</p> <p>3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p> <p>4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic.⁴</p> <p>1. Use place value understanding to round whole numbers to the nearest 10 or 100.</p> <p>2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.</p>	<p>Generalize place value understanding for multi-digit whole numbers.</p> <p>1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 + 70 = 10$ by applying concepts of place value and division.</p> <p>2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>3. Use place value understanding to round multi-digit whole numbers to any place.</p> <p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Understand the place value system.</p> <p>1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.</p> <p>2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <p>3. Read, write, and compare decimals to thousandths.</p> <p>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.</p> <p>b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p> <p>4. Use place value understanding to round decimals to any place.</p> <p>Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>5. Fluently multiply multi-digit whole numbers using the standard</p>

Use Place Value Understanding ...

Grade 1	Grade 2	Grade 3
<p>Use place value understanding and properties of operations to add and subtract.</p> <p>4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p> <p>Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p> <p>5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p> <p>6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models</p>	<p>Use place value understanding and properties of operations to add and subtract.</p> <p>5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p> <p>7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p> <p>8. Mentally add 10 or 100 to a given number 100–900, and mentally</p>	<p>Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>1. Use place value understanding to round whole numbers to the nearest 10 or 100.</p> <p>2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.</p>

Standards Progression

- To support analysis of standards across grades, the progressions within domains are being elaborated by the CCSS writers
- See <http://commoncoretools.wordpress.com>

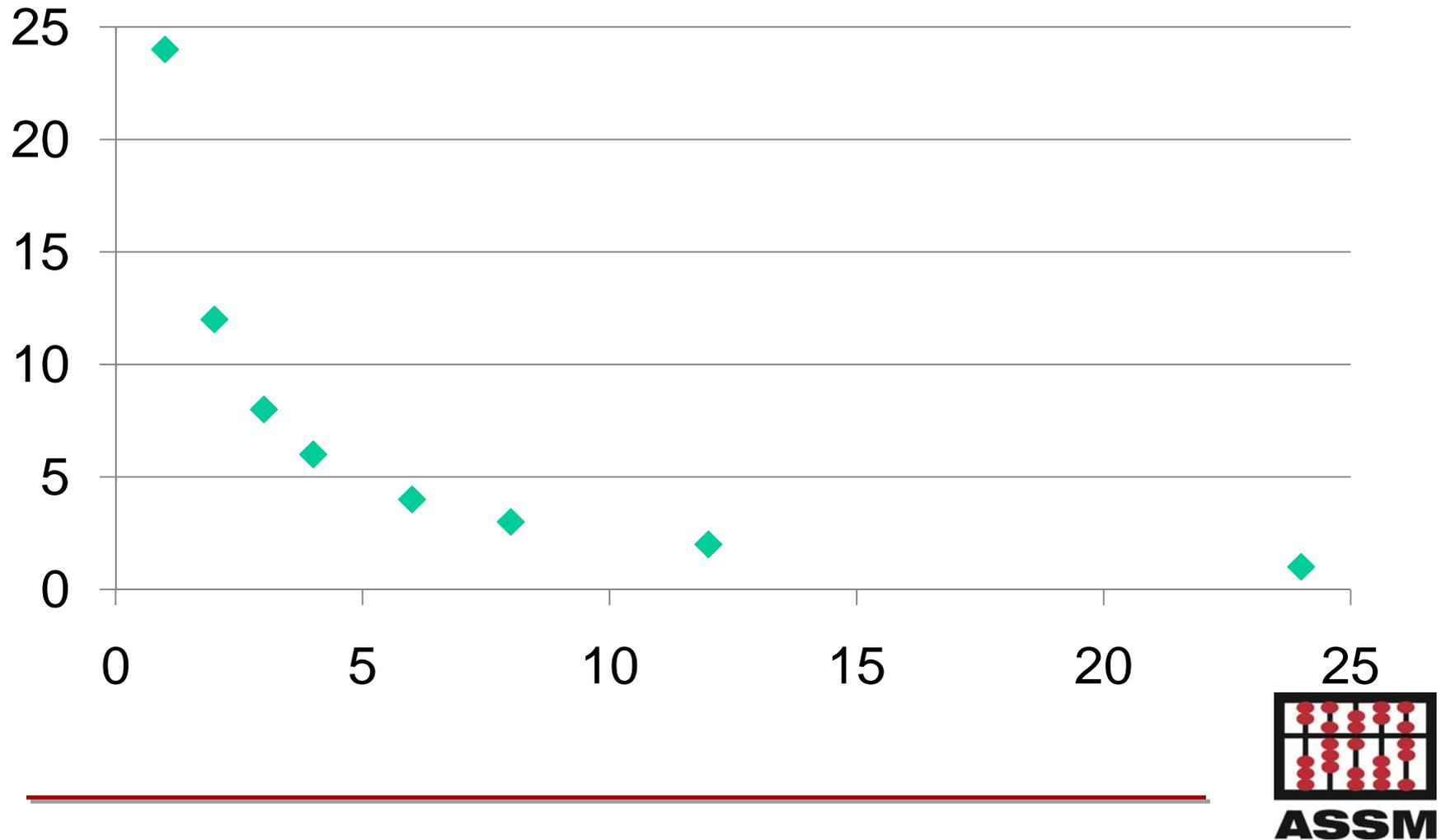
Task Progression

- A rich mathematical task can be reframed or resized to serve different mathematical goals
 - goals might lie in different domains

Constant Area, Changing Perimeter

- You have been asked to put together the dance floor for your sister's wedding. The dance floor is made up of 24 square tiles that measure one meter on each side.
 - Experiment with different rectangles that could be made using all of these tiles
 - Record your data in a table and a graph
 - Look for patterns in the data

Width vs. Length



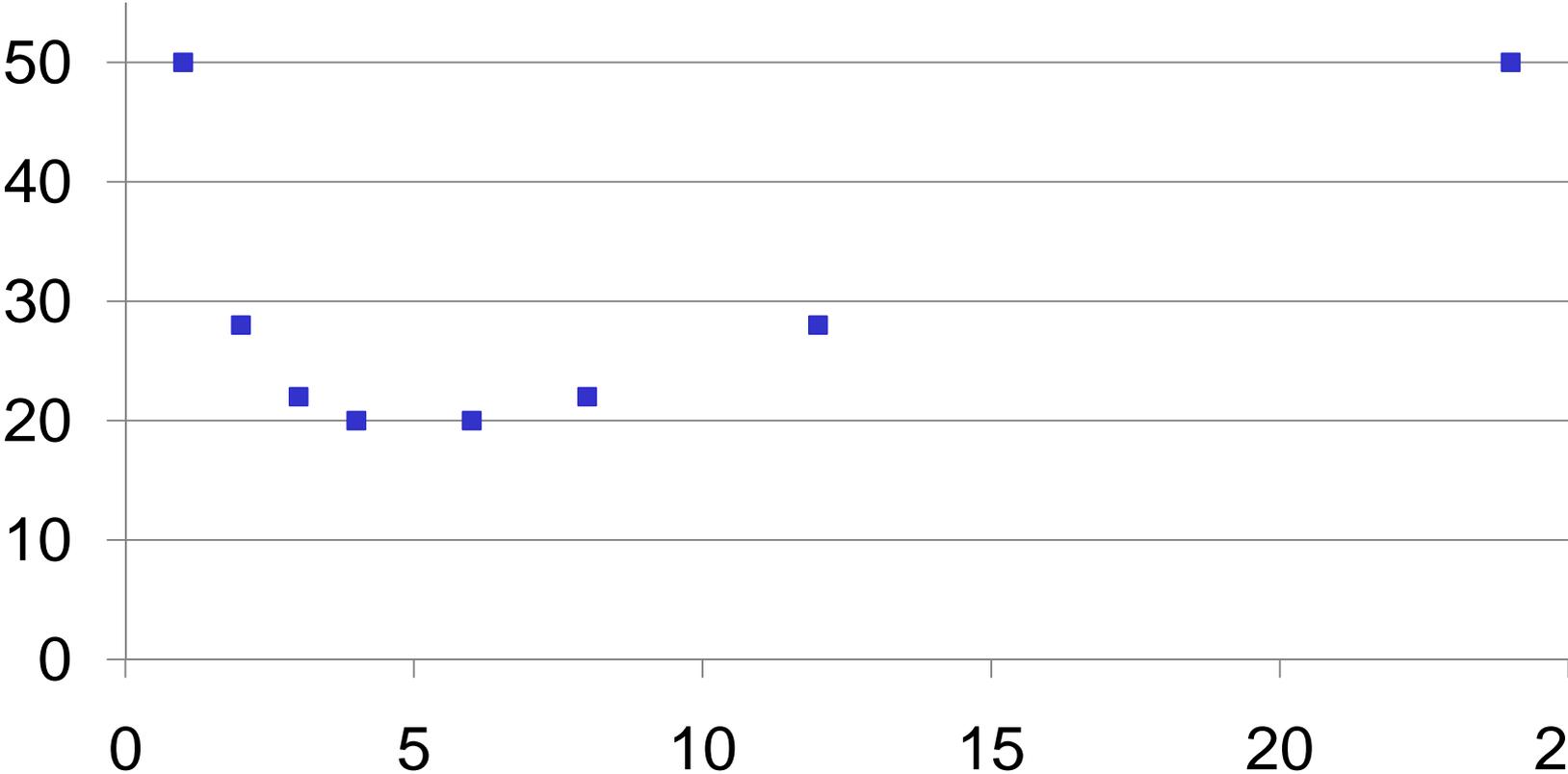
- Suppose the dance floor is held together by a border made of edge pieces one meter long.
 - What determines how many edge pieces are needed: area or perimeter? Explain.

Perimeter vs. Length

- Make a graph showing the perimeter vs. length for various rectangles with an area of 24 square meters.
- Describe the graph. How do patterns that you observed in the table show up in the graph?
- Which design would require the most edge pieces? Explain.
- Which design would require the fewest edge pieces? Explain.



Perimeter vs. Length

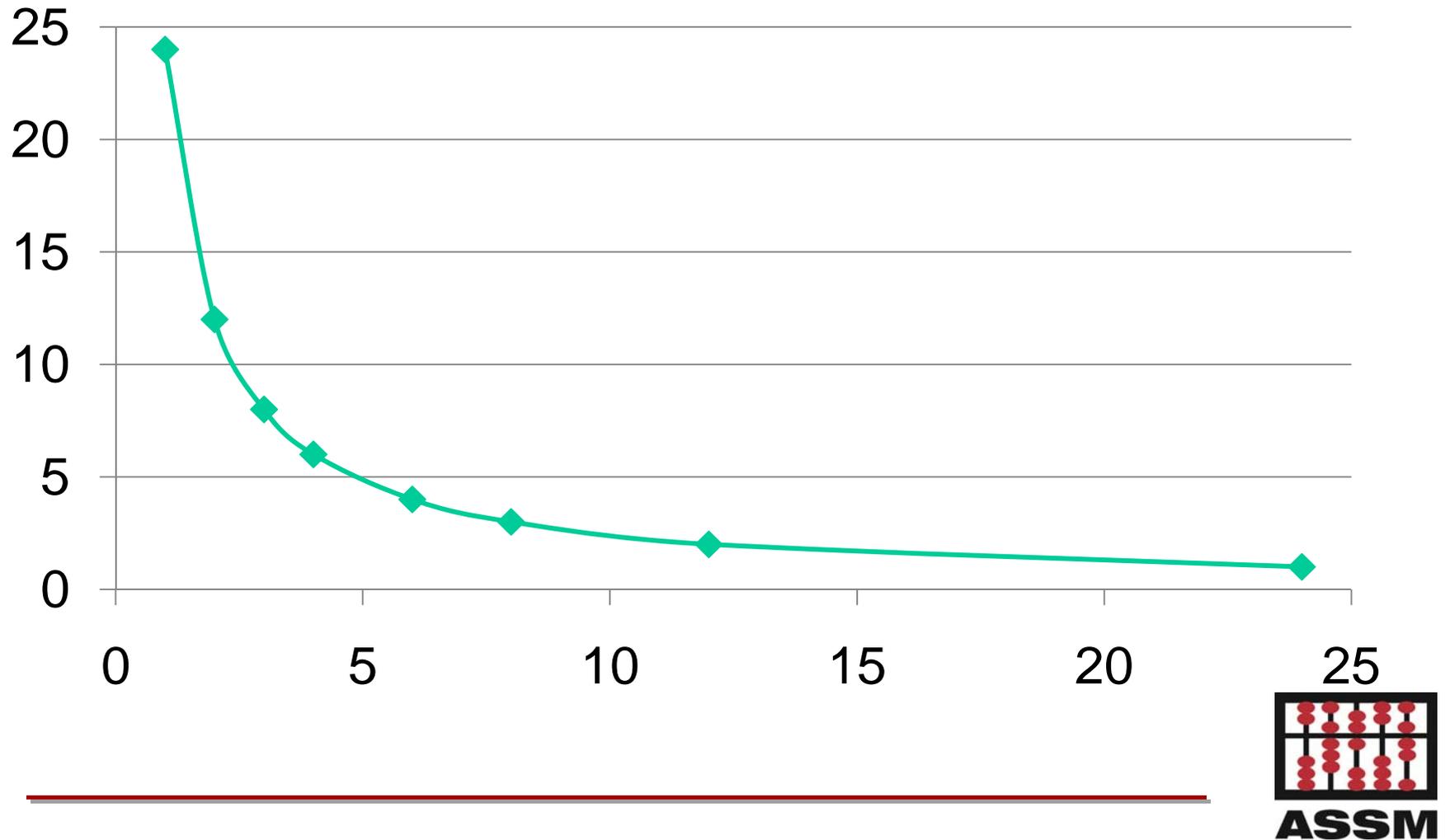


- Suppose you wish to design a dance floor using 36 square tiles that measure one meter on each side. Which design has the least perimeter? Which design has the greatest perimeter? Explain your reasoning.
- In general, describe the rectangle with whole-number dimensions that has the greatest perimeter for a fixed area. Which rectangle has the least perimeter for a fixed area?

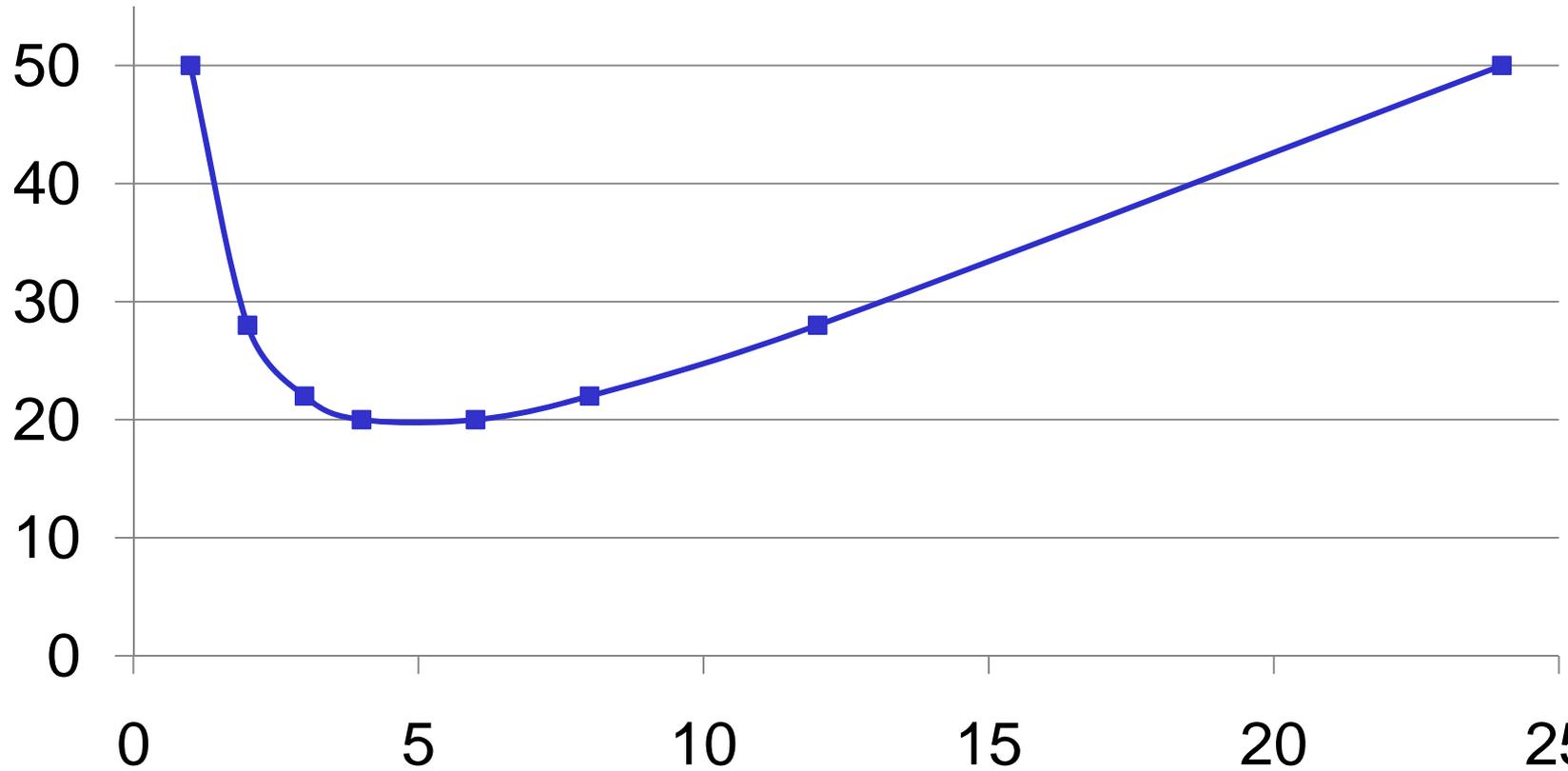
Extension Questions

- Can we connect the dots? Explain.
- How might we change the context so that the dimensions can be other than whole numbers?
- How would the previous answers change?

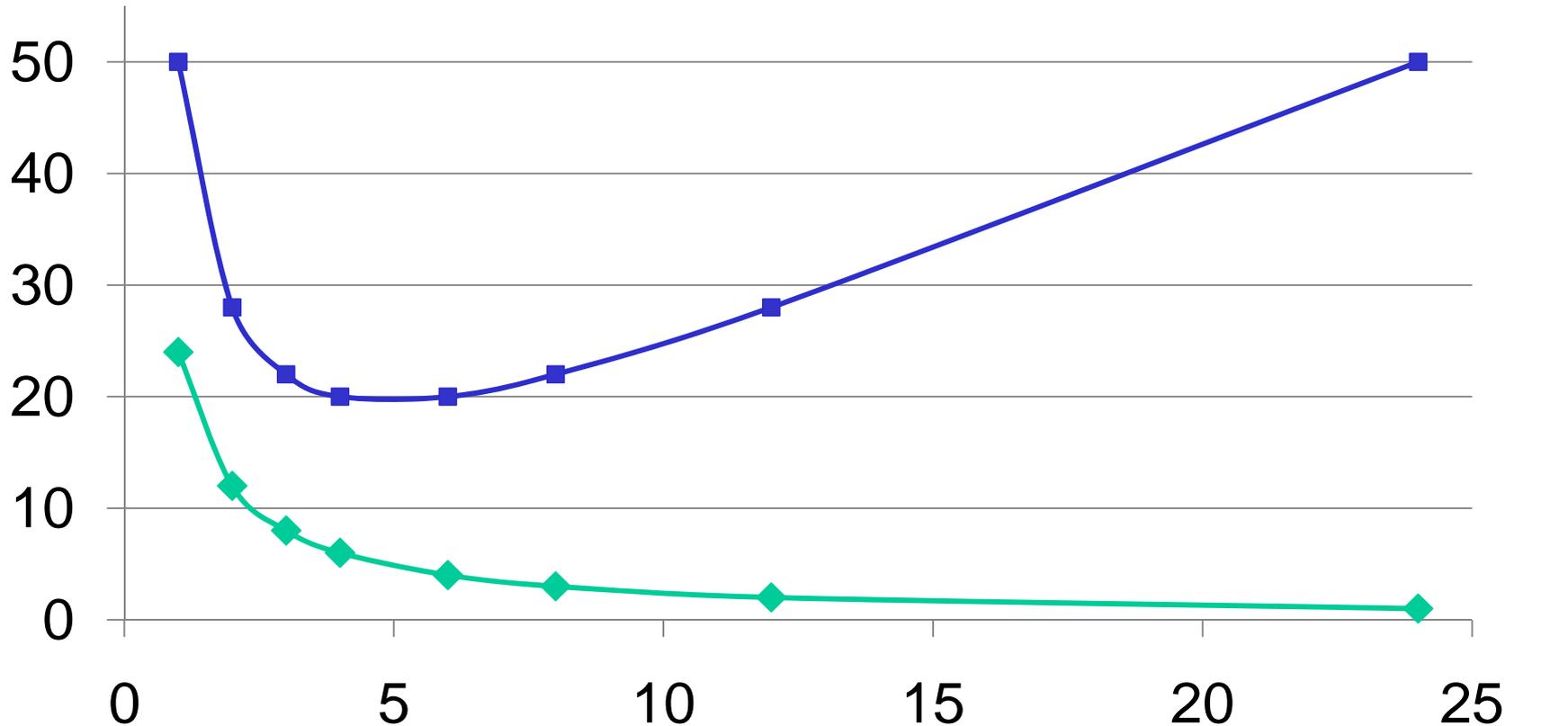
Width vs. Length



Perimeter vs. Length



Perimeter and Width vs. Length



Questions for Teachers

- How might we use this context and related contexts to support the learning at the level of Algebra 2 or its Equivalent (A2E)?
 - Domain and range
 - Limiting cases
 - Intercepts and asymptotic behavior?
 - Rates of change, maxima and minima
 - Equation solving with several variables?
 - Generalizing from a specific to a generic fixed quantity?

Perimeter and Area of Rectangles

- Fix one and vary the other
 - Grade 5: to distinguish the two quantities
 - Grade 9: to represent the quantities algebraically and to use graphs, tables, and formulas to explore how they are related
 - Grade 11: to distinguish linear, quadratic, and rational functions, and to explore domains in context and to push toward limiting cases
 - Calculus: as an optimization context in which to use differentiation
- Later, in multivariable calculus, explore relations among 3 or more variables



Progressions

- Learning progressions
 - Based in research on student learning
- Standards progressions
 - Built into standards
- Task progressions
 - Afforded by tasks

Connections

- How might these ideas help you think about
 - Formative Assessment
 - Differentiated Instruction
 - Response to Intervention
- For slides, see <http://www.assm.us>
- For examples of task that provide a ramp for access, see <http://insidemathematics.org>

