Implementing

the

Common Core State Standards

for

Mathematics

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Bradford R. Findell, PhD
brad@findell.org

Overview of the Day

- Background on the Common Core State Standards
- Working as
  - Students
  - Teachers
- Digging into the Common Core State Standards for
  - Mathematical content
  - Mathematical practices
  - Progressions

Major Themes

- All students means ALL students
- The work is about improving instruction, which requires that teachers collaborate to reach more students more of the time

CCSS Instructional Shifts

- **Focus**: focus strongly on key ideas, understandings, and skills in each grade and course
- **Coherence**: think across grades, and link to major topics in each grade
- **Rigor**: in major topics, pursue with equal intensity
  - conceptual understanding,
  - procedural skill and fluency, and
  - applications

CCSS Mathematical Practices

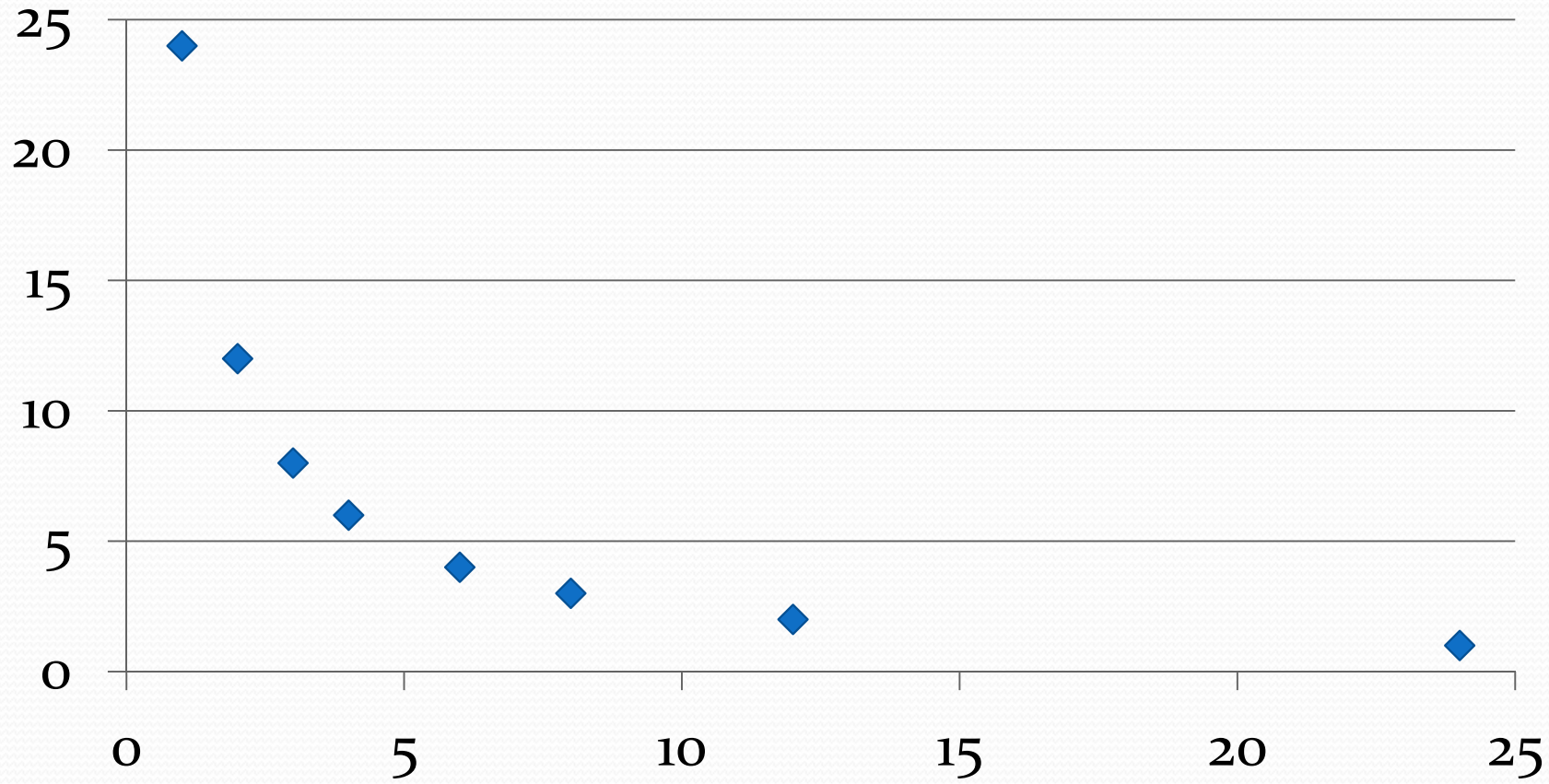
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Task Progression

Constant Area, Changing Perimeter

- You have been asked to put together the dance floor for your sister's wedding. The dance floor is made up of 24 square tiles that measure one meter on each side.
  - Experiment with different rectangles that could be made using all of these tiles
  - Record your data in a table and a graph
  - Look for patterns in the data

Width vs. Length

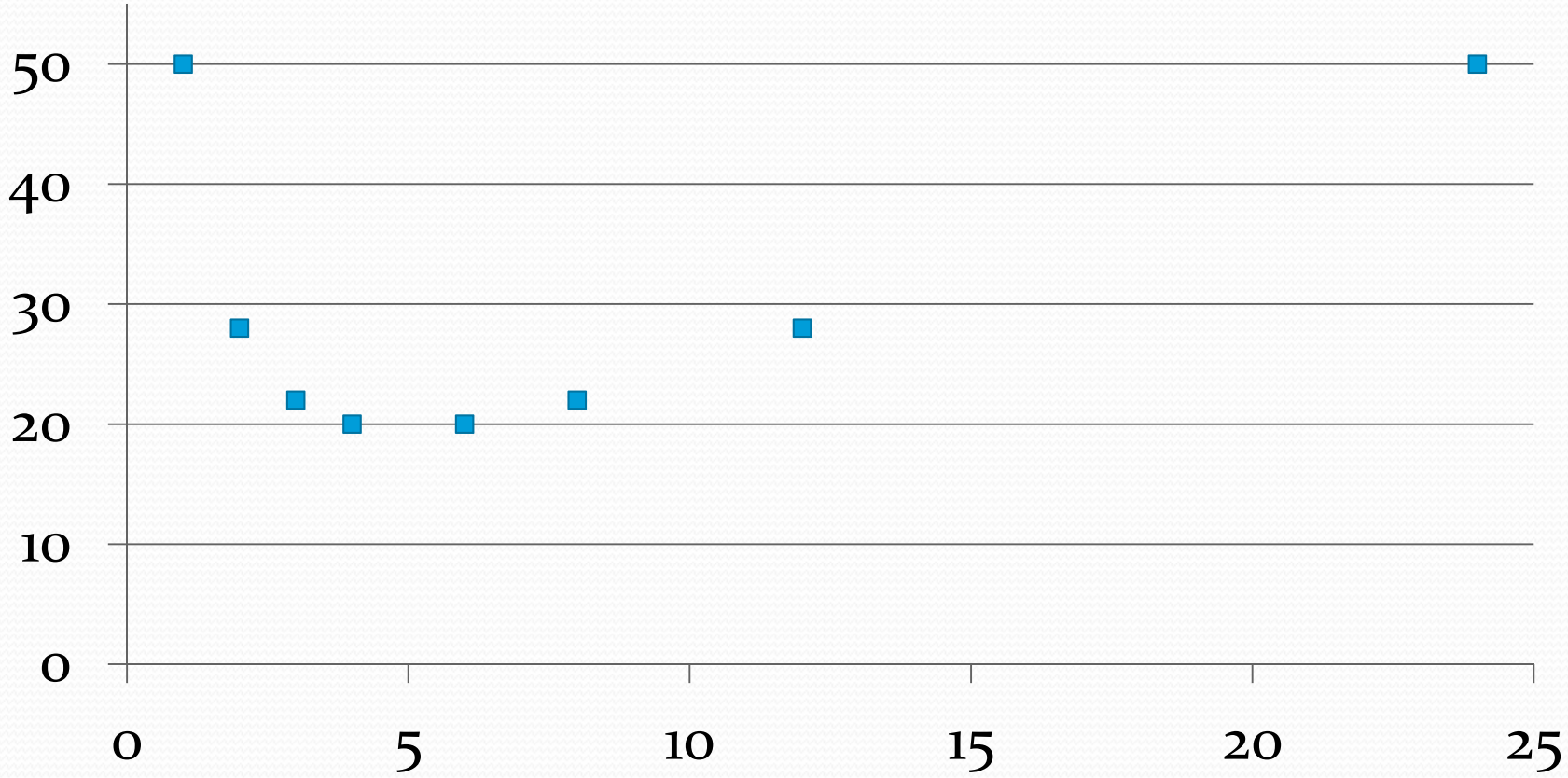


- Suppose the dance floor is held together by a border made of edge pieces one meter long.
  - What determines how many edge pieces are needed: area or perimeter? Explain.

Perimeter vs. Length

- Make a graph showing the perimeter vs. length for various rectangles with an area of 24 square meters.
- Describe the graph. How do patterns that you observed in the table show up in the graph?
- Which design would require the most edge pieces? Explain.
- Which design would require the fewest edge pieces? Explain.

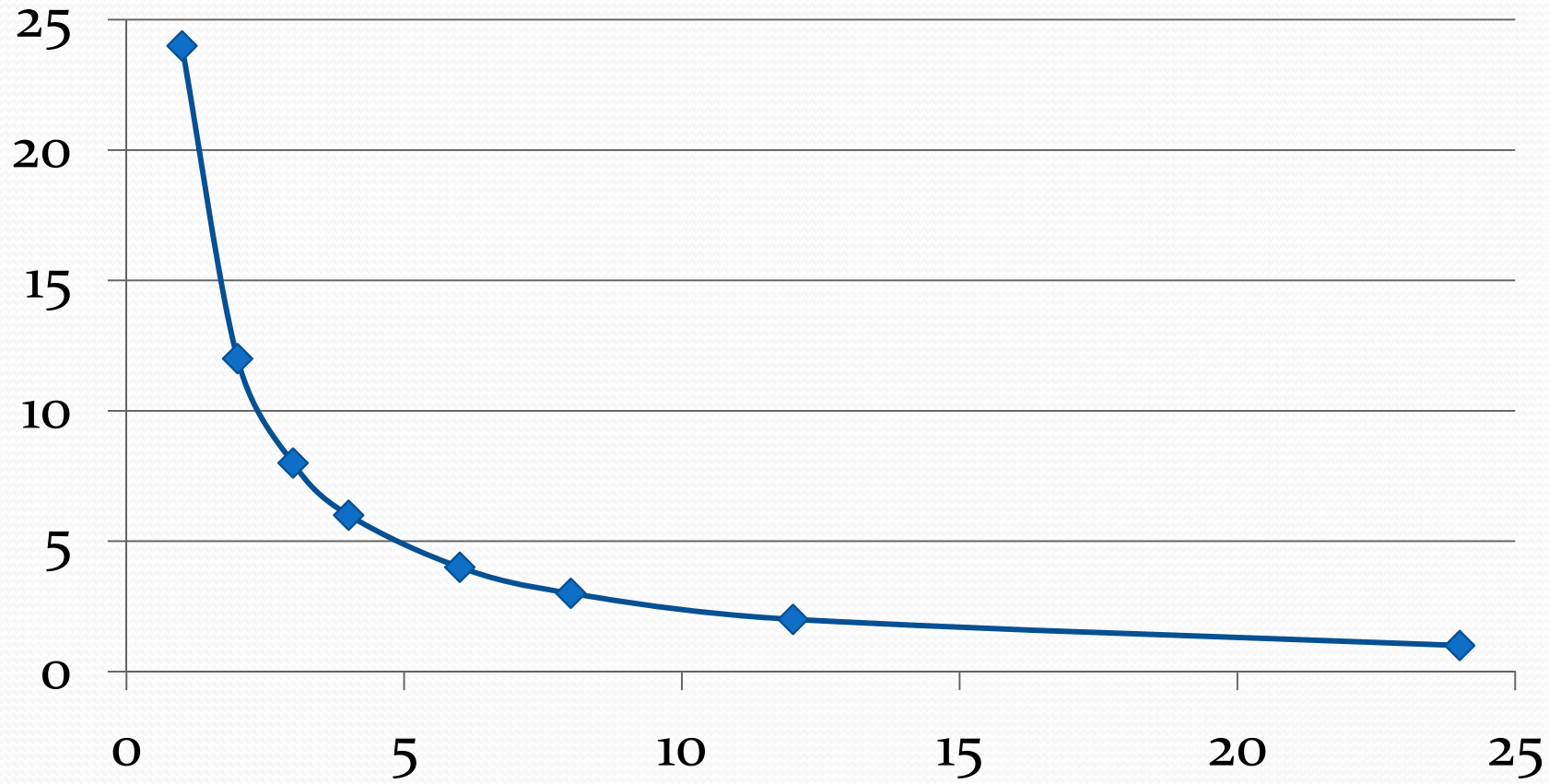
Perimeter vs. Length



Extension Questions

- Can we connect the dots? Explain.
- How might we change the context so that the dimensions can be other than whole numbers?
- How would the previous answers change?
- In general, describe the rectangle with whole-number dimensions that has the greatest perimeter for a fixed area. Which rectangle has the least perimeter for a fixed area?

Width vs. Length



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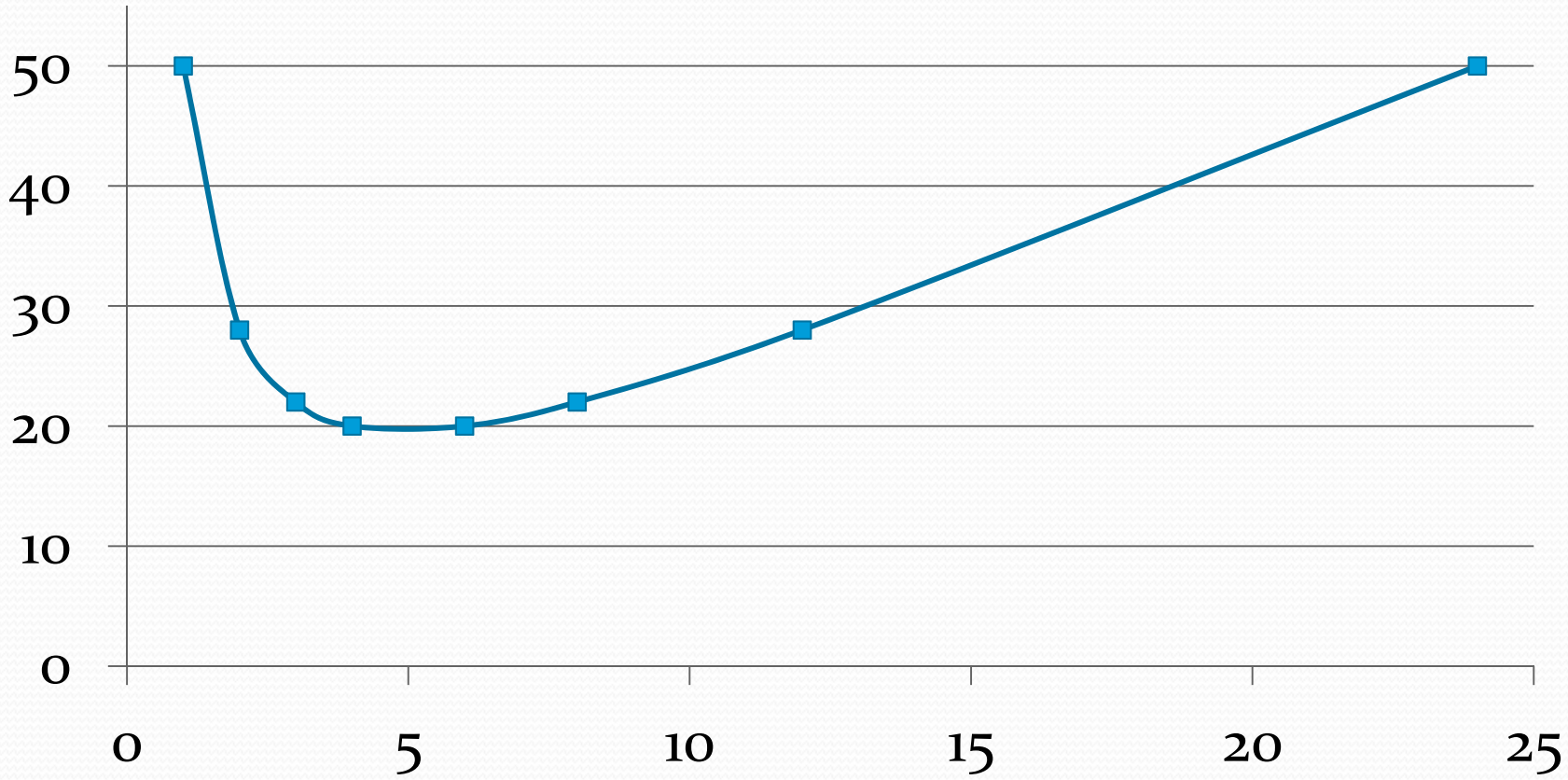
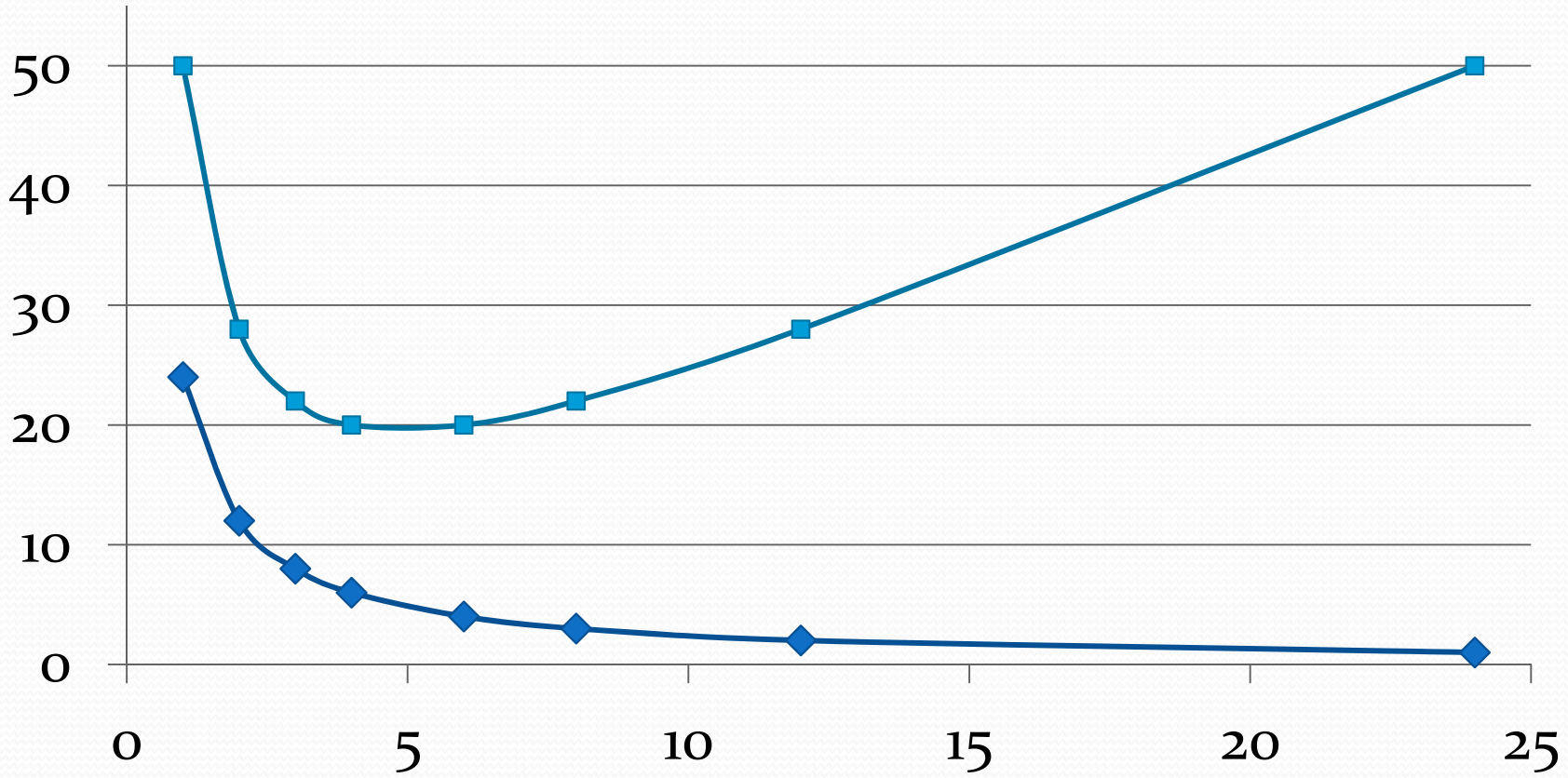


Perimeter and Width vs. Length



Questions for Teachers

- How might we use this context and related contexts to support the learning at the level of Algebra 2 or its Equivalent (A2E)?
  - Domain and range
  - Limiting cases
  - Intercepts and asymptotic behavior?
  - Rates of change, maxima and minima
  - Equation solving with several variables?
  - Generalizing from a specific to a generic fixed quantity?

Perimeter and Area of Rectangles

- Fix one and vary the other
  - Grade 3: to distinguish the two quantities
  - Grade 5: to plot ordered pairs to see relationships
  - Grade 8: to represent the quantities algebraically and to use graphs, tables, and formulas to explore how they are related
  - Grade 11: to distinguish linear, quadratic, and rational functions, and to explore domains in context and to push toward limiting cases
  - Calculus: as an optimization context in which to use differentiation
- Later, in multivariable calculus, explore relations among 3 or more variables

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Considerations

- How might these ideas help you think about
  - Formative Assessment
  - Differentiated Instruction
  - Response to Intervention

A Real-World Problem

- One weekend, I was gathering sticks from my lawn, bundling them in each hand. When both hands became full, I found that by using both hands for a single bundle, I could gather quite a few more sticks. Why?
- What relationship should I expect between the quantities of sticks gatherable by the two methods?

Sketch of Solution

- The capacity to gather sticks depends on the cross-sectional area of the circles made by my fingers. That area varies with the square of circumference. So two hands together have four times the capacity of one hand, or two times the capacity of both hands separately.
- *Note: With algebra, we can prove this more precisely*.

Qualitative Shifts in Content

Pythagorean Theorem

- 8.G.6. Explain a proof of the Pythagorean theorem and its converse.
- 8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- G-SRT.4. Prove theorems about triangles. *Theorems include … the Pythagorean Theorem proved using triangle similarity.*
- G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem …
- F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

- *The Pythagorean Theorem is not just "$a^2 + b^2 = c^2$."*

Sequences as Functions

- F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
- F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- *Emphasize connections among patterns, sequences, and functions.*

Rules of Exponents

- 8.EE.1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

- *These ideas support many HS standards on exponential functions.*

Solving Equations

- 8.EE.7.a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- A-REI.A. Understand solving equations as a process of reasoning and explain the reasoning
- A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$;

- *This last standard supports the many different techniques for solving different types of equations.*

Seeing Structure in Expressions

- A-SSE.1. Interpret expressions that represent a quantity in terms of its context.
- A-SSE.2. Use the structure of an expression to identify ways to rewrite it.
- A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- *"Simplest form" depends on the purpose.*

Rational and Irrational Numbers

- 7.NS.2.d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in os or eventually repeats.
- 8.NS.1. Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in os or eventually repeat. Know that other numbers are called irrational.
- N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

- *This content is seldom taught.*

Can You Provide Three Explanations?

- Why is division by 0 undefined? Is $0/0 = 1$?
- Why is $a^0 = 1$? And does it matter what a is?
- Why is $a^{-n} = 1/a^n$? And does it matter what a is?
- Is 0 even, odd, neither, or both?
- Why is a negative times a negative positive?
- When multiplying fractions, why do we multiply numerators and denominators?
- When dividing by a fraction, why is it okay to invert and multiply?
- Is $0.9999\ldots = 1$?

Toward Focus and Coherence

Focusing Secondary Courses

- Organize by course
  - Grade 7, Grade 8, Algebra 1, Geometry, or Algebra 2
- Questions to drive discussion:
  - What can we leave out?
  - What should we focus on?
- Summarize your findings

- What about review?
  - Instead, use previous content in service of new ideas
- What about remediation?
  - Provide access to the regular curriculum AND extra support