

# Algebra 2 for All?

## How Much Is Enough?

Westerville City Schools  
November 3, 2015

Bradford R. Findell, PhD  
[findell.2@osu.edu](mailto:findell.2@osu.edu)

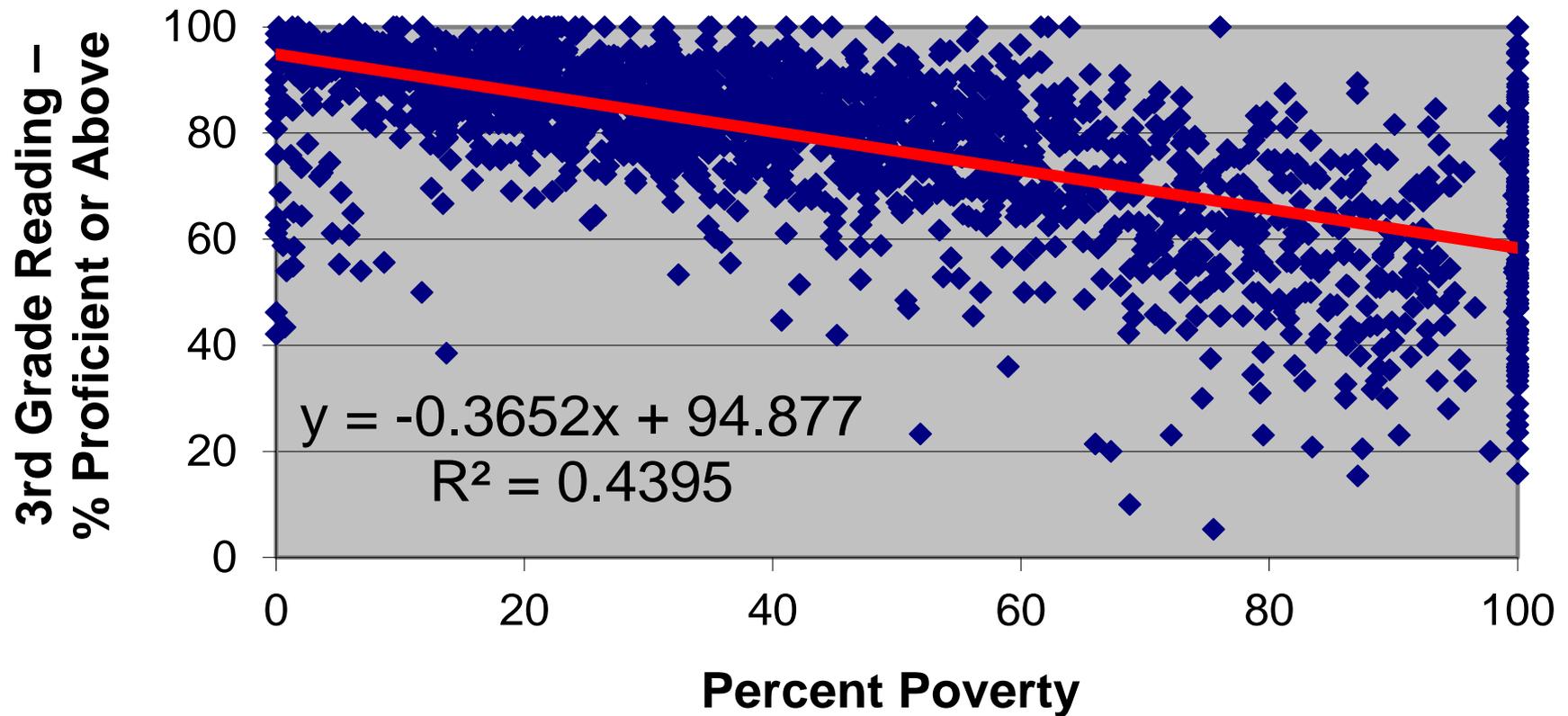


# College and Career Readiness

- College and career readiness involves mathematics at the level of Algebra 2 or its equivalent (A2E)
- All students need proficiency in A2E for
  - Many careers, with or without college
  - Informed citizenship
  - Individual empowerment
- High school mathematics should open doors
  - But adult decisions often close doors for students
  - After students complete A2E, they have choices
- But not your parents' Algebra 2

# Who Can Interpret This?

**SY2006-07 - 3rd Grade Reading and Percent Poverty  
by School**



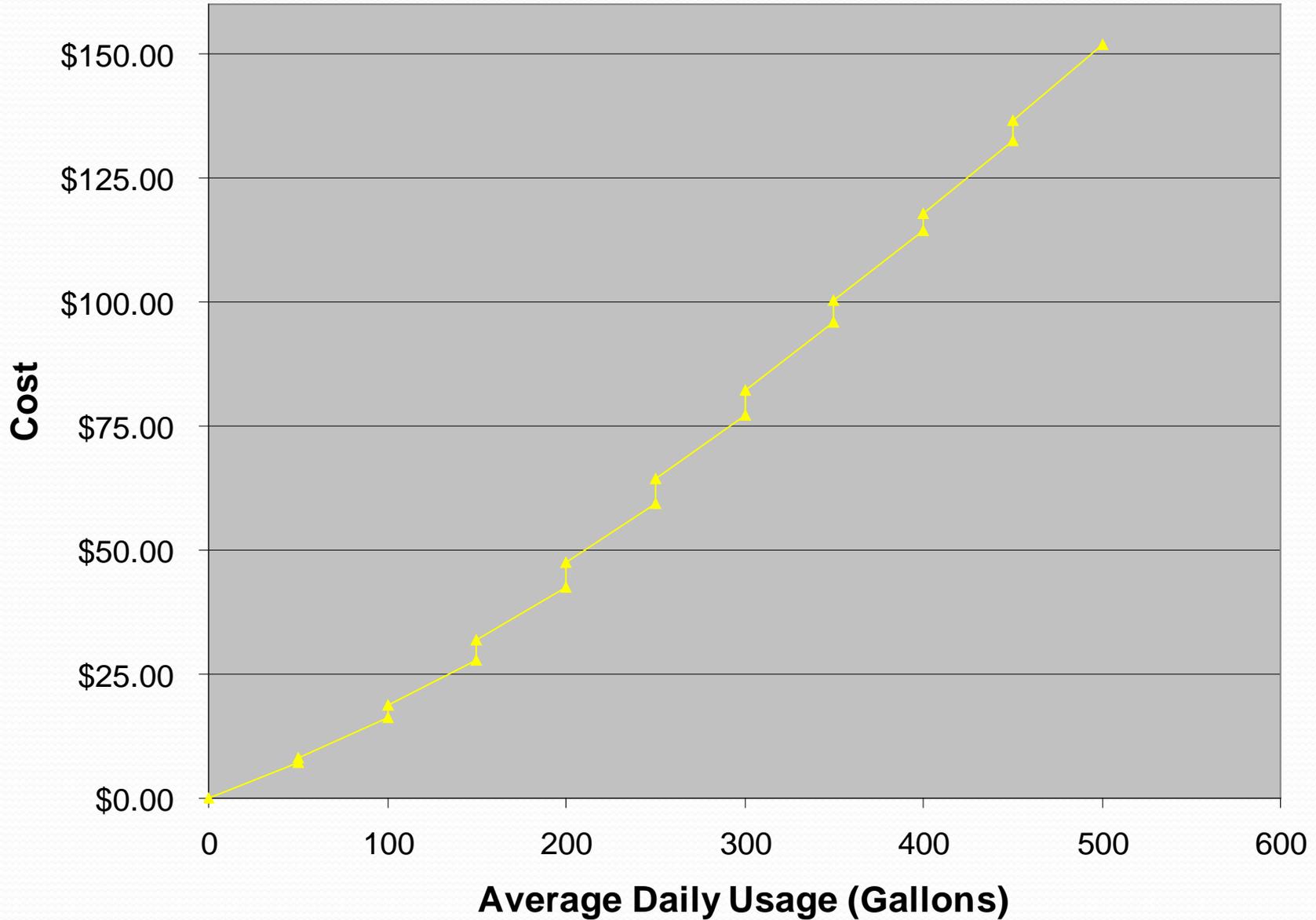
# Washington Suburban Sanitary Commission

## Rate Schedule, July 1, 2008

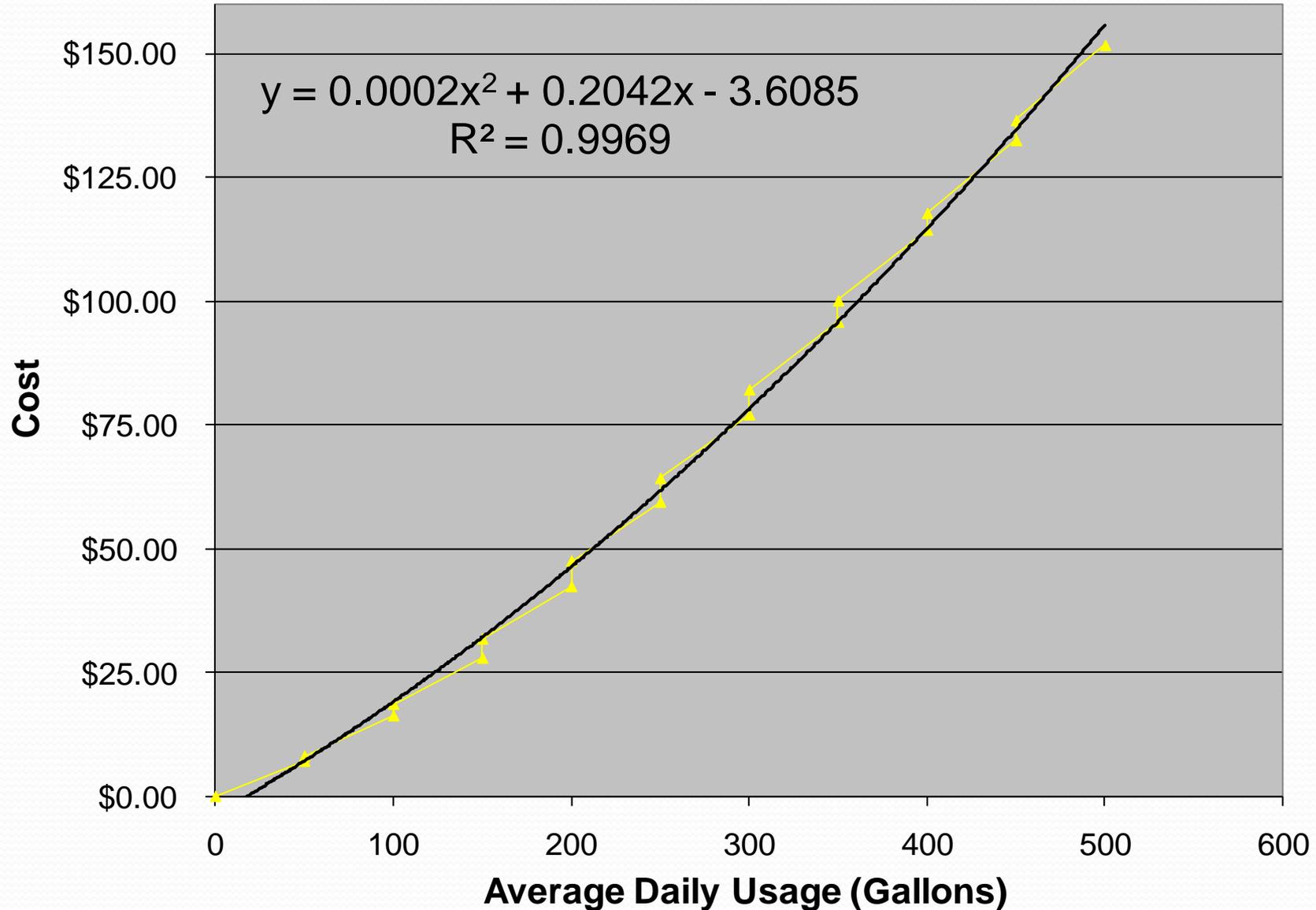
<b>Average Daily Consumption (Gallons/Day)</b>	<b>Water Rate Per 1,000 Gallons</b>	<b>Sewer Rate Per 1,000 Gallons</b>	<b>Combined Rate Per 1,000 Gallons</b>
0-49	\$1.97	\$2.77	\$4.74
50 - 99	2.21	3.22	5.43
100 - 149	2.42	3.79	6.21
150 - 199	2.71	4.36	7.07
200 - 249	3.17	4.76	7.93
250 - 299	3.43	5.14	8.57
300 - 349	3.63	5.50	9.13
350 - 399	3.79	5.75	9.54
400 - 449	3.94	5.88	9.82
...	...	...	...

Source: <http://www.wsscwater.com/service/rates.cfm>

# Monthly Water and Sewer Cost



# Monthly Water and Sewer Cost





# What Is Needed?

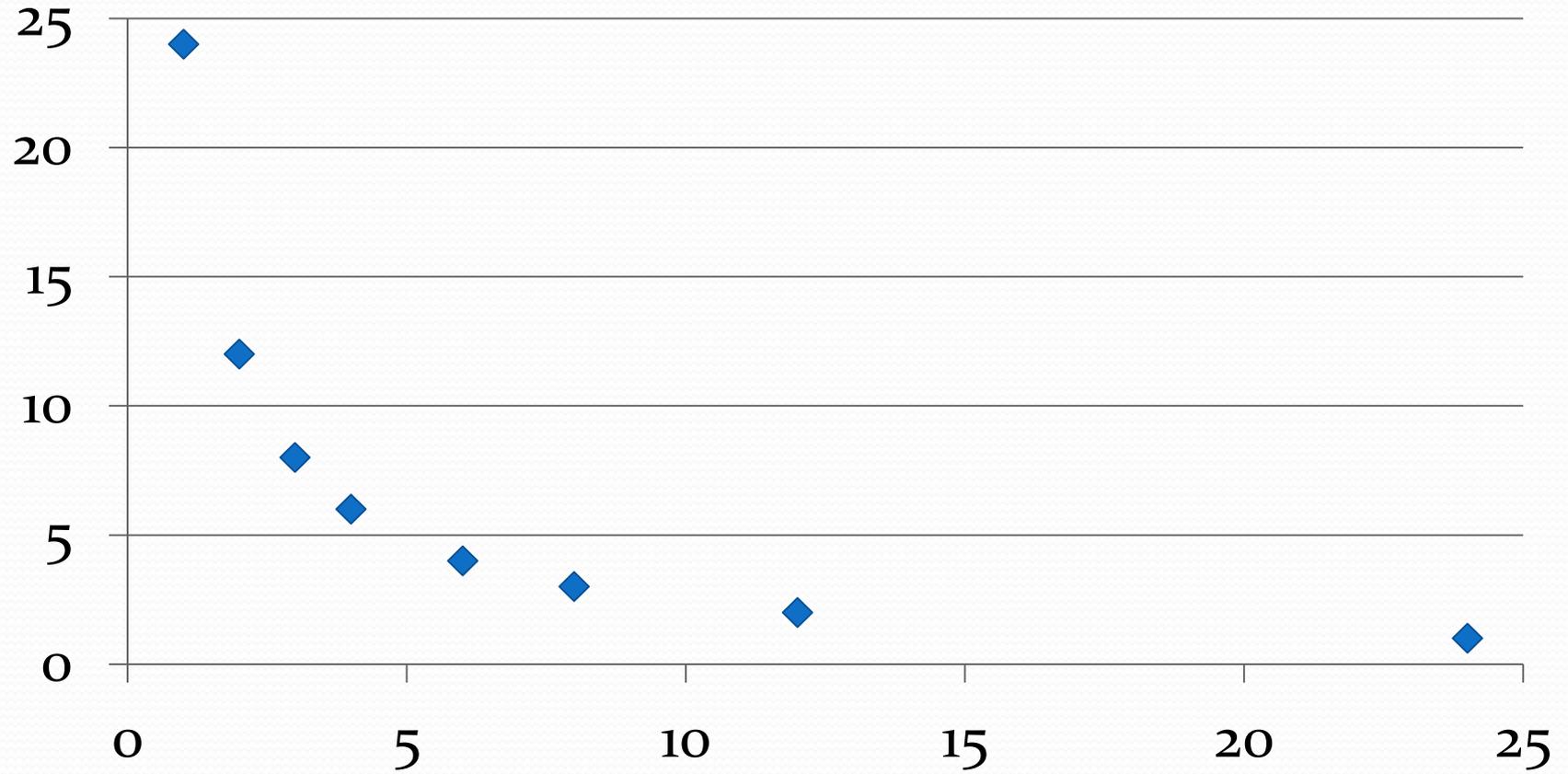
- Renewed curriculum and instruction
  - Especially across middle and high school, toward a rigorous, relevant, and accessible A2E
- Support for students are behind
  - To help them catch up
- The CCSS, the Model Pathways, and new curricular materials are foundational responses to these needs

# Task Progression

# Constant Area, Changing Perimeter

- You have been asked to put together the dance floor for your sister's wedding. The dance floor is made up of 24 square tiles that measure one meter on each side.
  - Experiment with different rectangles that could be made using all of these tiles
  - Record your data in a table and a graph
  - Look for patterns in the data

# Width vs. Length

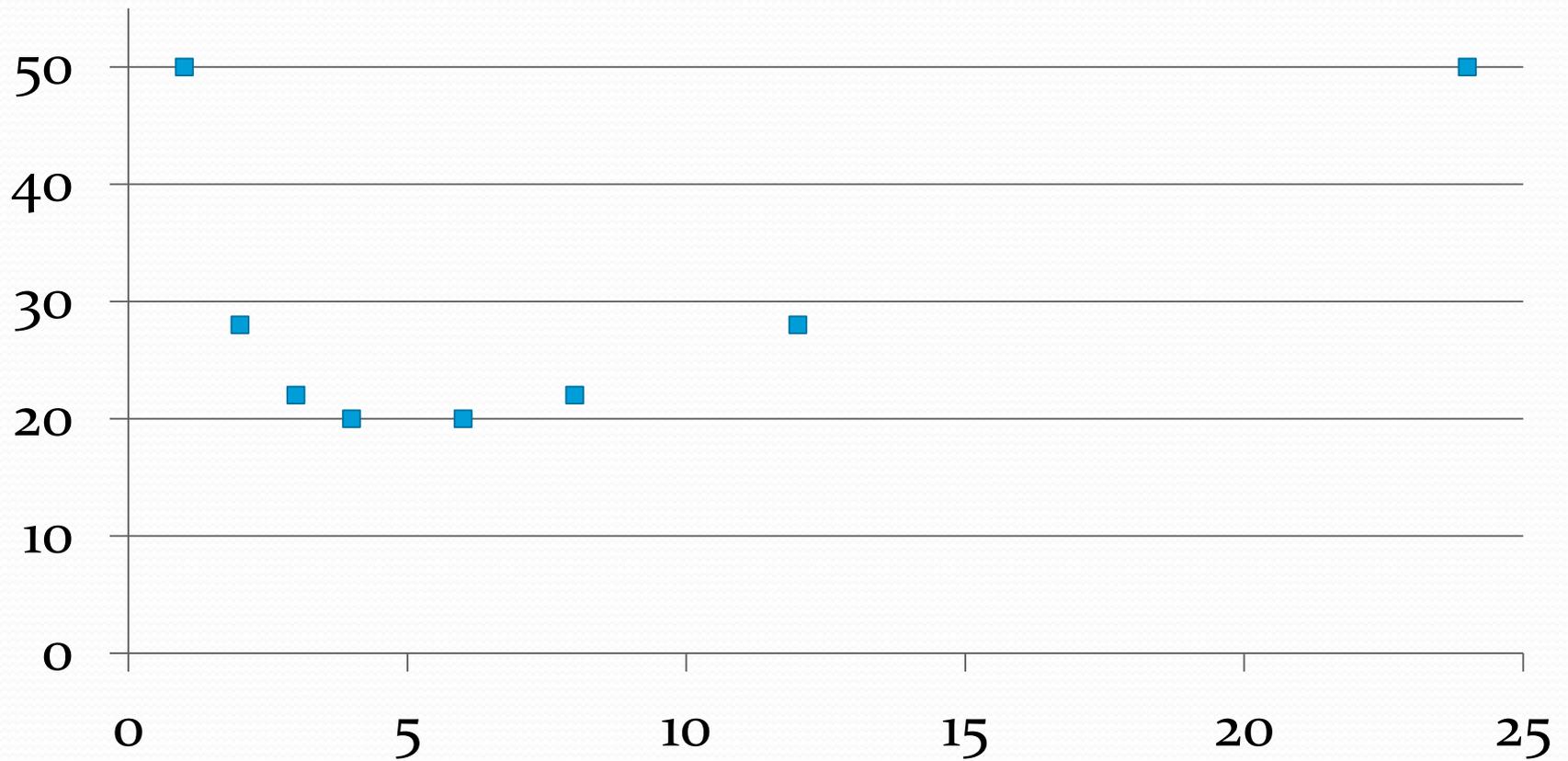


- 
- Suppose the dance floor is held together by a border made of edge pieces one meter long.
  - What determines how many edge pieces are needed: area or perimeter? Explain.

# Perimeter vs. Length

- Make a graph showing the perimeter vs. length for various rectangles with an area of 24 square meters.
- Describe the graph. How do patterns that you observed in the table show up in the graph?
- Which design would require the most edge pieces? Explain.
- Which design would require the fewest edge pieces? Explain.

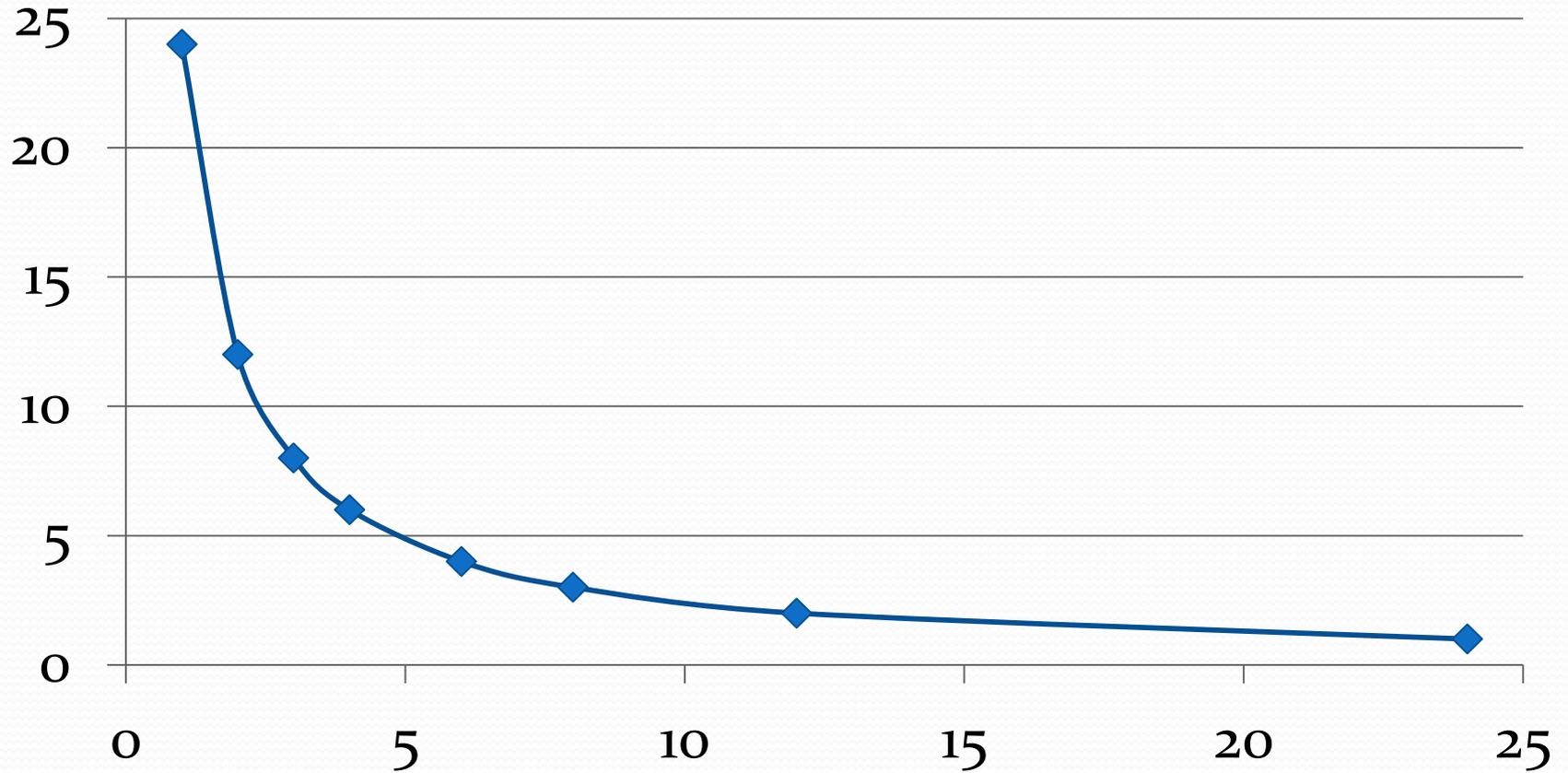
# Perimeter vs. Length



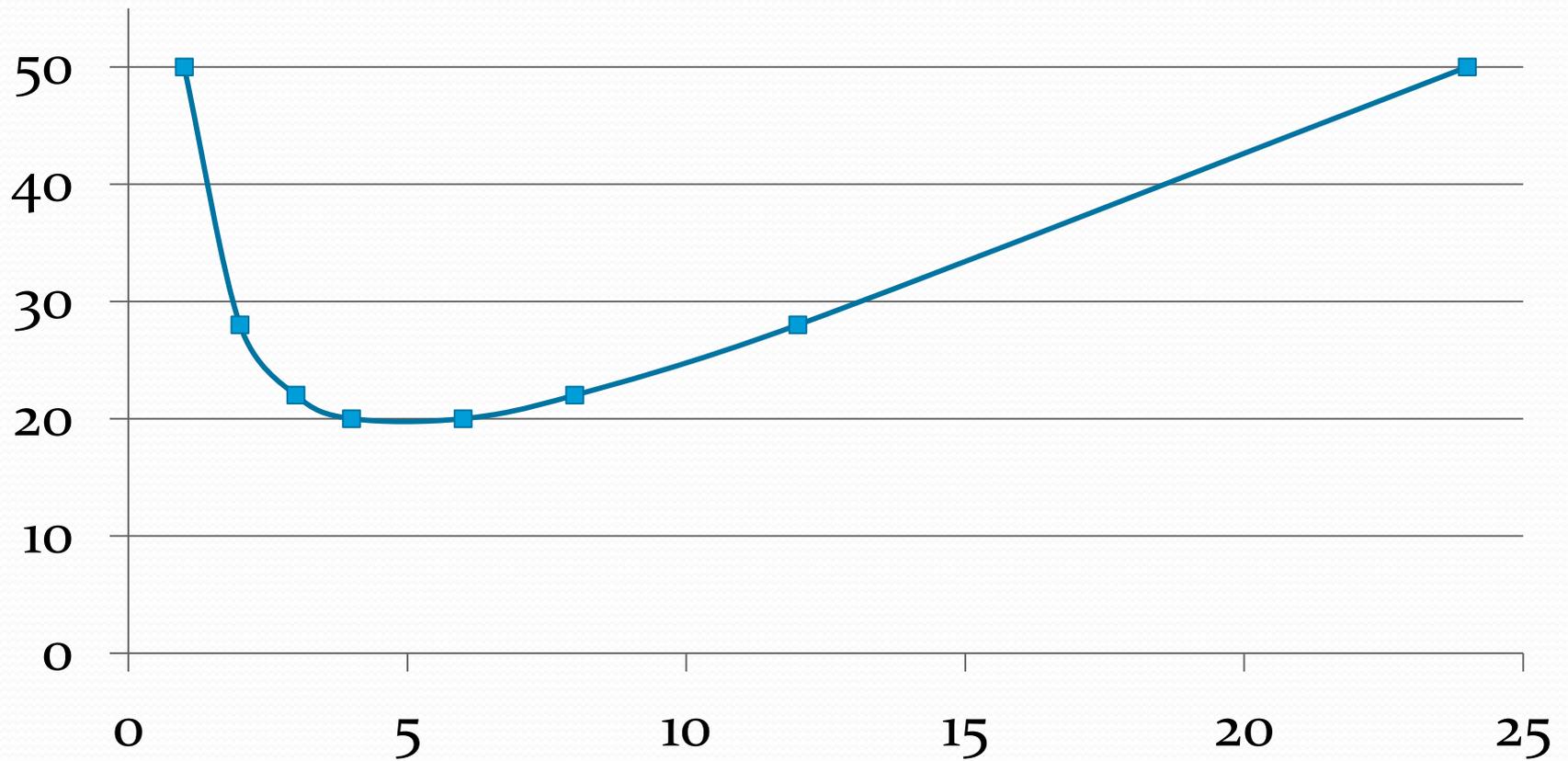
# Extension Questions

- Can we connect the dots? Explain.
- How might we change the context so that the dimensions can be other than whole numbers?
- How would the previous answers change?
- In general, describe the rectangle with whole-number dimensions that has the greatest perimeter for a fixed area. Which rectangle has the least perimeter for a fixed area?

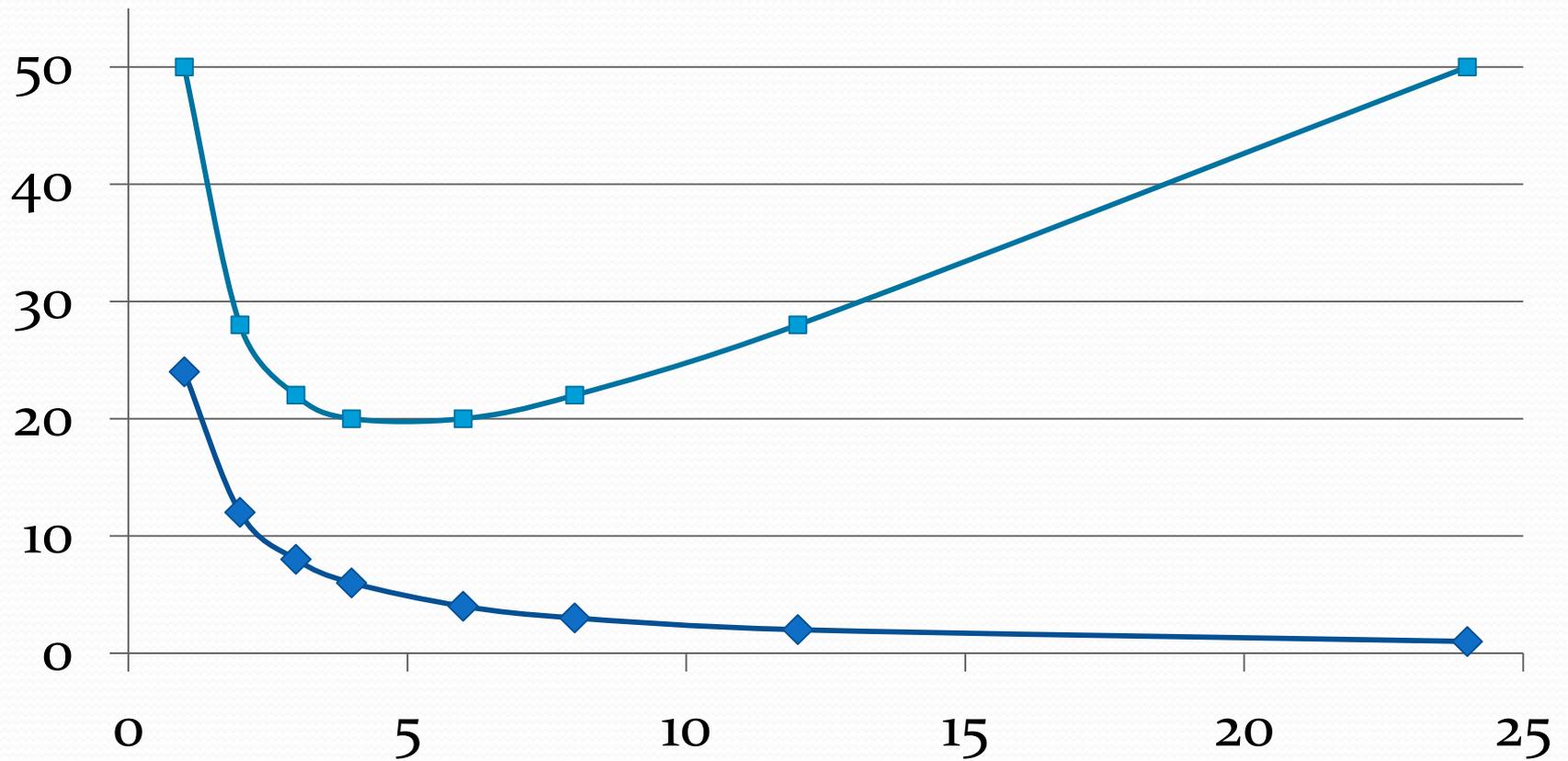
# Width vs. Length



# Perimeter vs. Length



# Perimeter and Width vs. Length



# Related Problems

- What if we fix the perimeter?
  - Explore width vs. length
  - Explore area vs. length
- What if we fix the width?
  - Explore area vs. length
  - Explore perimeter vs. length
- Explore these functions in Geogebra
  - <http://tube.geogebra.org/material/show/id/978091>
  - What kinds of functions are these?
    - Explain graphically, symbolically, in tables, and in context



# Questions for Teachers

- How might we use this context to support the learning at the level of A2E?
  - Domain and range
  - Limiting cases
  - Intercepts and asymptotic behavior?
  - Rates of change, maxima and minima
  - Equation solving with several variables?
  - Generalizing from a specific to a generic fixed quantity?

# Perimeter and Area of Rectangles

- Fix one and vary the other
  - Grade 3: to distinguish the two quantities
  - Grade 5: to plot ordered pairs to see relationships
  - Grade 8: to represent the quantities algebraically and to use graphs, tables, and formulas to explore how they are related
  - Grade 11: to distinguish linear, quadratic, and rational functions, and to explore domains in context and to push toward limiting cases
  - Calculus: as an optimization context in which to use differentiation
- Later, in multivariable calculus, explore relations among 3 or more variables

# CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

# Considerations

- How might these ideas help you think about
  - Formative Assessment
  - Differentiated Instruction
  - Response to Intervention

# Toward Focus and Coherence in A2E

# Sources Consulted for Content of A2E

- Content of Intermediate (non-credit) and College Algebra at OSU
- Ohio Transfer Module for College Algebra
  - <https://www.ohiohighered.org/transfer/transfermodule/learningoutcomes>
  - <http://regents.ohio.gov/transfer/otm/math-stats-log/TMM001-College-Algebra.pdf>
- Research on learning algebra and precalculus
  - [https://mathed.asu.edu/about/marilyn\\_carlson.shtml](https://mathed.asu.edu/about/marilyn_carlson.shtml)
- PARCC Model Content Frameworks (for Algebra 2 and Math 3)
  - [http://www.parcconline.org/sites/parcc/files/PARCC\\_MCF\\_Mathematics-12-11-2014.pdf](http://www.parcconline.org/sites/parcc/files/PARCC_MCF_Mathematics-12-11-2014.pdf)
- PARCC tests for Algebra 2
  - <http://parconline.org/take-the-test>
- ACT score/standards
  - <http://www.act.org/standard/planact/math/index.html>
- Experience with preservice middle grades teachers at OSU

# Key Ideas for A2E

- Rates of change
- Modeling
  - Direct and inverse proportions
  - Linear, quadratic, and exponential functions
- General thinking
  - Thinking generally with specific numbers
  - Seeing specific examples in general statements
  - Seeing structure in expressions: recognizing the “form” of an expression
- Rule of four: numerically, symbolically, graphically, and verbally (in context)

# Key Ideas for A2E

- Distinguishing expressions, equations, and functions
  - Expressions have values
  - Equations have solutions
  - ...
- Meaning of notation
  - order of operations, function notation, absolute value, exponents, fraction
  - Mathematician's law of the repeated variable.
- Important connections
  - Exponential functions, exponents, radicals
  - Number of solutions

# Reminders

- What has been called “Algebra 1” begins in Grade 8 for all students
  - High school Algebra should build on Grade 8
- Universities want students using algebra every year
  - Geometry courses cannot be a “year off” from algebra