Fractions to Functions: Algebra Readiness Through Proportional Reasoning

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Stacking Paper

- Suppose you want to know how many sheets are in a particular stack of paper, but don't want to count the pages directly. You have the following information:
  - The given stack has height 4.50 cm.
  - A ream of 500 sheets has height 6.25 cm.
- How many sheets of paper do you think are in the given stack?

- From Stanley, 2014. See more at:
  - http://blogs.ams.org/matheducation/2014/11/20/proportionality-confusion/

Mixing Punch

- Jenny is mixing punch and is considering two recipes:
  - Recipe A: 3 parts orange juice for every 5 parts ginger ale
  - Recipe B: 2 parts orange juice for every 3 parts ginger ale
- Which recipe will give juice that is the most "orangey"?
  - Explain your reasoning.
- Use a table to show various ways to make recipe B
- To make 12 gallons of recipe B, how much of each will you need?

Racing Snails

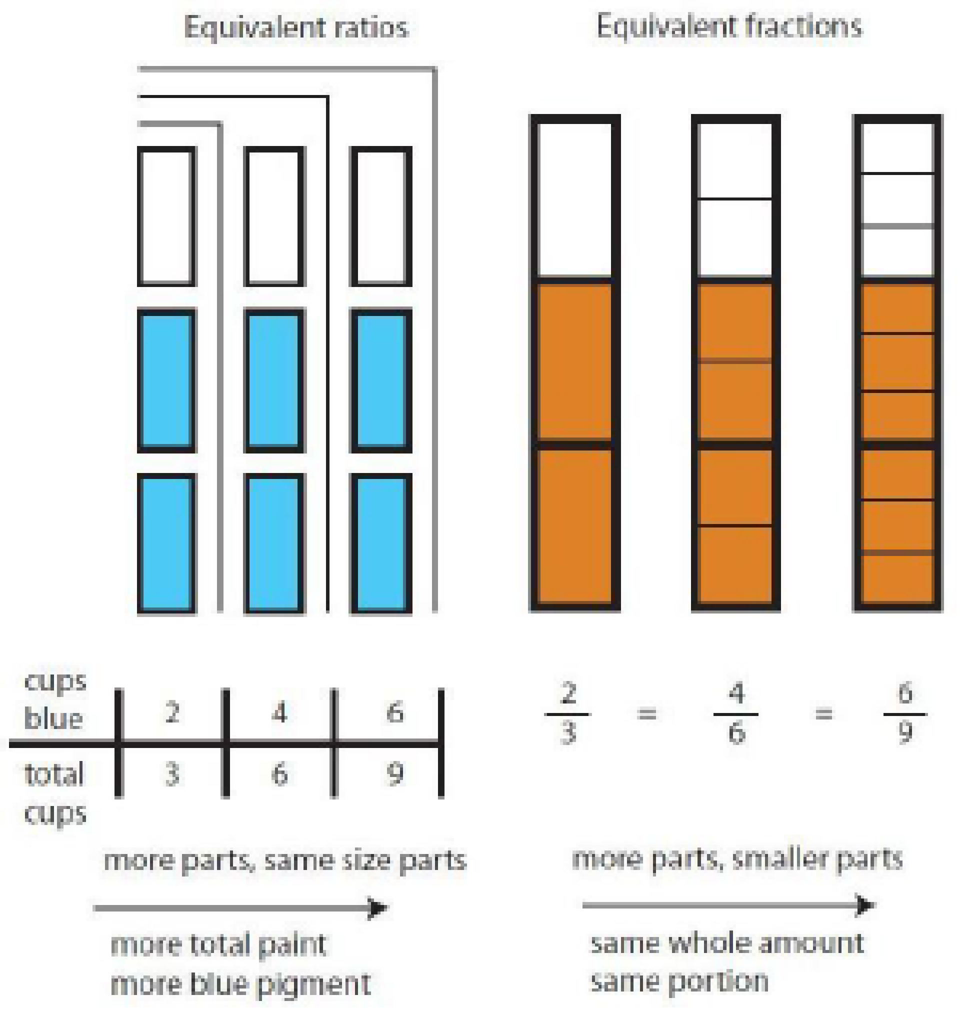
- Mike is racing snails who move at a constant speed:
  - Snail A travels 3 inches in 5 minutes
  - Snail B travels 2 inches in 3 minutes
- Which snail moves faster? Explain your reasoning.
- Use a table to show other distances and times for snail B.

Tools for Proportional Reasoning

- Equivalent fractions
- Equivalent ratios
- Ratio tables
- Unit rates
- Double number lines
- Graphs

- Following are pictures from *Draft 6-7 Progression on Ratios and Proportional Relationships*, available at
  - http://math.arizona.edu/~ime/progressions/

Equivalent ratios versus equivalent fractions



Three ways to compare paint mixtures

Same amount of red

Abby's

cups red	cups yellow
1	3
2	6
3	9
4	12
5	15

Zack's

cups red	cups yellow
3	5
6	10
9	15
12	20
15	25

Same amount of red.
Abby's has more yellow,
so Abby's is yellower,
Zack's is redder.

Same amount of yellow

Abby's

cups red	cups yellow
1	3
2	6
3	9
4	12
5	15

Zack's

cups red	cups yellow
3	5
6	10
9	15
12	20
15	25

Same amount of yellow.
Zack's has more red.
So Zack's is redder.
Abby's is yellower.

Same total

Abby's

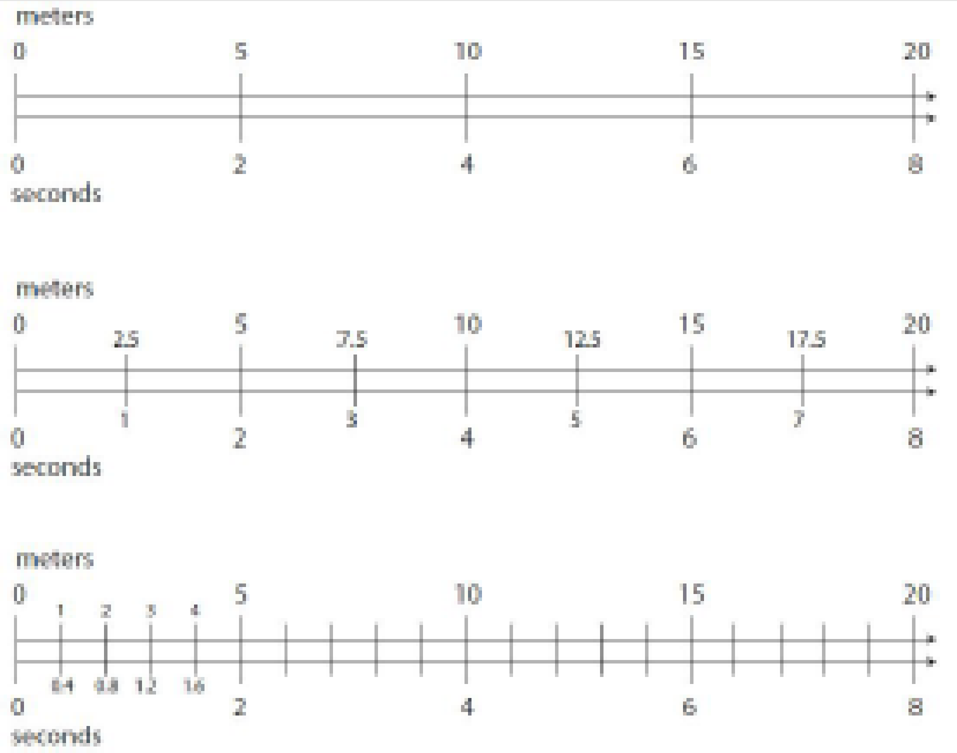
cups red	cups yellow	total cups
1	3	4
2	6	8
3	9	12

Zack's

cups red	cups yellow	total cups
3	5	8
6	10	16
9	15	24

Same total.
Abby's has more yellow.
Zack's has more red.
So Abby's is yellower and
Zack's is redder.

Double number line diagrams used for situations with different units



Double number lines indicate coordinated multiplying and dividing of quantities. This can also be indicated in tables.

Showing structure in tables and graphs

Additive Structure

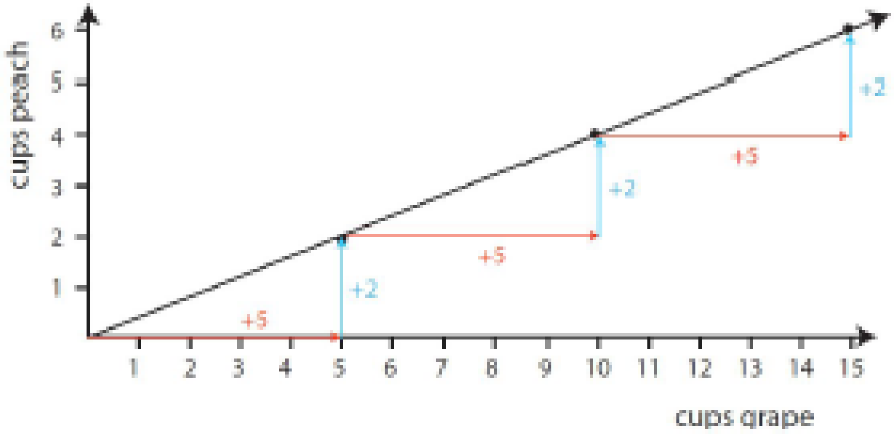
cups grape	cups peach
5	2
10	4
15	6
20	8
25	10

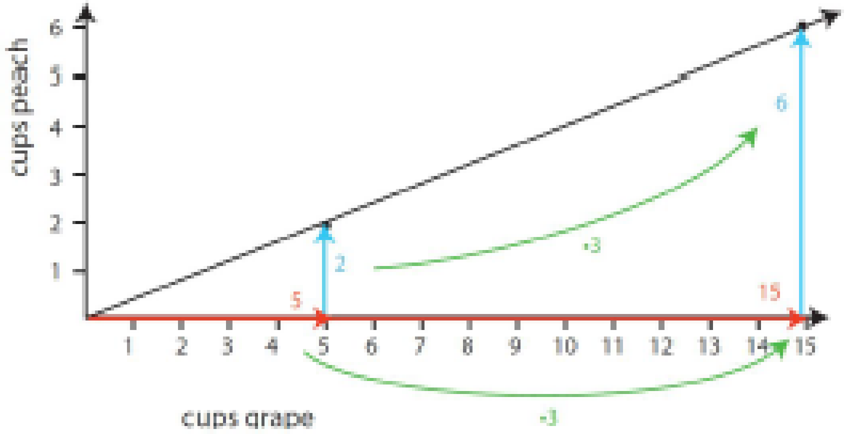
+5 +5 +5 +5 (grape column); +2 +2 +2 +2 (peach column)

Multiplicative Structure

cups grape	cups peach
5	2
10	4
15	6
20	8
100	40

·2 ·3 ·20 (each column)





In the tables, equivalent ratios are generated by repeated addition (left) and by scalar multiplication (right). Students might be asked to identify and explain correspondences between each table and the graph beneath it (MP1).

Big Ideas in Grade 8

- Linear functions:
  - directly proportional relationships vs. linear functions that are not
- Compare to functions that are not linear

Fractions, Ratios, and Rates

- Three connected ideas with differing histories and differing usage
  - Fractions are numbers, often used to express results of sharing, cutting
  - Ratios have historically been used to compare “like” quantities
    - Often expressed as pairs of counting numbers, without units, e.g., 3:2
  - Rates have historically been used to compare different quantities
    - Often expressed as quotients of quantities (e.g., meters and seconds)
- Ultimately, we want students to see all of these as quotients (i.e., the result of a division)
  - In high school and beyond, the distinctions become blurred
- Sometimes it is useful to attend only to the numbers
  - Two apparently different problems might be abstractly “the same”
- It is important to use the units to interpret numeric “answers” in context

CCSS Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

MP2. Reason abstractly and quantitatively

- Mathematically proficient students make sense of **quantities and their relationships** in problem situations.
- They bring two complementary abilities to bear on problems involving quantitative relationships:
  - the ability to ***decontextualize***—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—
  - and the ability to ***contextualize***, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

MP2. Reason abstractly and quantitatively (cont.)

- Quantitative reasoning entails habits of
  - **creating a coherent representation** of the problem at hand;
  - **considering the units** involved;
  - **attending to the meaning of quantities**, not just how to compute them; and
  - knowing and flexibly using different properties of operations and objects.

MP5. Model with mathematics

- Mathematically proficient students can **apply the mathematics they know to solve problems** arising in everyday life, society, and the workplace.
  - In early grades, this might be as simple as writing an addition equation to describe a situation.
  - In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later.

MP5. Model with mathematics (cont.)

- They are able to **identify important quantities** in a practical situation **and map their relationships** using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- They can **analyze those relationships** mathematically to draw conclusions.
- They routinely **interpret their mathematical results** in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematical Modeling (simplified)