



Ohio's Mathematics Standards Revision: Implications for Grades K-5

Bradford R. Findell, PhD

Westerville Educator Day

August 14, 2017



History of Common Core State Standards

- 2007: Governors and state superintendents began discussing shared standards across interested states
- 2009: Common Core State Standards Initiative launched
 - 48 states participated in the development of standards in ELA and mathematics
 - Broad, bipartisan effort
- 2010: Common Core State Standards (CCSS) released
 - Ohio adopted the Common Core in June 2010 without modification
 - By late 2011, Common Core adopted by 45.5 states and the District of Columbia
 - A few states later dropped CCSS; other states “customized” CCSS



Ohio's Standards Revision Infrastructure

- Public comment
 - Organized by standard
 - Called for claims, resolutions, and research/rationale
- Advisory Committee
 - Representatives of stakeholder organizations
 - Findell represented the Ohio Department of Higher Education
 - Reviewed public comments, provided directives for working groups
- Working Groups
 - Grade bands: K-5, 5-9, High School
- Ohio Department of Education staff



Ohio's Standards Revision Timeline, 2016

- March: Standards posted for public comment
- April and May: Advisory Committee
- May and June: Working Groups
- July: Proposed revisions posted for public comment
- August and September: Working Groups and Advisory Committee
- October to December: Presentation to Legislature and State Board of Education
- February, 2017: Approval by State Board of Education



Revision Process, by the Numbers

- 385 mathematics standards reviewed online
- 242 standards received 647 public comments
- 100 standards and 10 additional directives forwarded to Working Groups
- 155 changes to standards
 - Most changes were about clarity, embedding footnotes, or “clarity by HS course”
 - 13 standards with proposed revisions reverted to original
 - 41 standards with substantive changes, deleted, moved, or new
- 23 changes to cluster headings
 - 14 changes for clarity, including 7 embedded footnotes
 - 9 changes for vertical alignment, including 3 new clusters



Ohio Mathematics

- For more information, see the ODE Mathematics site:
 - <http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics>



Examples of Changes for Clarity



Clarifying Learning Targets

Original Standards	Revised Standards
<p>Know number names and the count sequence. K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).</p>	<p>Know number names and the count sequence. K.CC.2 Count forward within 100 beginning from any given number other than 1.</p>
<p>Know number names and the count sequence. K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).</p>	<p>Know number names and the count sequence. K.CC.3 Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).</p>
<p>K.CC.6 Compare numbers. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.</p>	<p>Compare numbers. K.CC.6 Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.</p>



Providing Helpful Detail

Original Standards	Revised Standards
<p>Understand place value. 2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.</p>	<p>Understand place value. 2.NBT.2 Count forward and backward within 1000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5.</p>
<p>Understand place value. 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p>	<p>Understand place value. 2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, expanded form, and equivalent representations, e.g., 716 is $700 + 10 + 6$, or $6 + 700 + 10$, or 6 ones and 71 tens, etc.</p>



Providing Cautionary Detail

Original Standards	Revised Standards
<p>Develop understanding of fractions as numbers. 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p>b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p>	<p>Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.</p> <p>3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.</p> <p>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.</p> <p>b. Represent a fraction a/b (which may be greater than 1) on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.</p>



Providing Too Much Detail?

Original Standards	Revised Standards
<p>Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>	<p>Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>5.NBT.7 <i>Solve real-world problems by adding, subtracting, multiplying, and dividing decimals</i> using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, <i>or multiplication and division</i>; relate the strategy to a written method and explain the reasoning used.</p> <p>a. <i>Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place).</i></p> <p>b. <i>Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place).</i></p> <p>c. <i>Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.</i></p>



Fending off Misinterpretation

Original Standards	Revised Standards
<p>Represent and solve problems involving multiplication and division.</p> <p>3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>For example, describe a context in which a total number of objects can be expressed as 5×7.</i></p>	<p>Represent and solve problems involving multiplication and division.</p> <p>3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. <i>(Note: These standards are written with the convention that $a \times b$ means a groups of b objects each; however, because of the commutative property, students may also interpret 5×7 as the total number of objects in 7 groups of 5 objects each).</i></p>



Conforming to Convention

Original Standards	Revised Standards
<p>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</p> <p>5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</p> <p>a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</p> <p>b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</p> <p>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</p>	<p>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</p> <p>5.MD.5 Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.</p> <p>a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication.</p> <p>b. Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.</p> <p>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.</p>



Examples of Substantive Changes



New Progression on Money

Original Standards	Revised Standards
<p>Count to tell the number of objects. K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality. [a,b,c]</p>	<p>Count to tell the number of objects. K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies. [a,b,c]</p>
<p>Tell and write time. 1.MD.3 Tell and write time in hours and half-hours using analog and digital clocks.</p>	<p>Work with time and money. 1.MD.3 Work with time and money. a. Tell and write time in hours and half-hours using analog and digital clocks. b. Identify pennies and dimes by name and value.</p>
<p>Work with time and money. 2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i></p>	<p>Work with time and money. 2.MD.8 Solve problems with money. a. Identify nickels and quarters by name and value. b. Find the value of a collection of quarters, dimes, nickels, and pennies. c. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and ¢ symbols appropriately (not including decimal notation).</p>



New Progression on Money (cont.)

Original Standards	Revised Standards
<p>Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</p> <p>3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>	<p>Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.</p> <p>3.MD.1 <i>Work with time and money.</i></p> <p>a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock.</p> <p>b. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the \$ and ¢ symbol appropriately (not including decimal notation).</p>
<p>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>	<p>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>4.MD.2 <i>Solve real-world problems involving money, time, and metric measurement.</i></p> <p>a. Using models, add and subtract money and express the answer in decimal notation.</p> <p>b. Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes.</p> <p>c. Add, subtract, and multiply whole numbers to solve metric</p>



Measurement Separation

Original Standards	Revised Standards
<p>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),...</i></p>	<p>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>4.MD.1 Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. <i>For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...</i></p>
<p>Convert like measurement units within a given measurement system.</p> <p>5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p>	<p>Convert like measurement units within a given measurement system.</p> <p>5.MD.1 Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.</p>



Order of Operations Postponed to Grade 6

Original Standards	Revised Standards
<p>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p> <p>3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>	<p>Solve problems involving the four operations, and identify and explain patterns in arithmetic.</p> <p>3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.</p>
<p>Write and interpret numerical expressions.</p> <p>5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</p>	<p>Write and interpret numerical expressions.</p> <p>5.OA.1 Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary.</p>



Order of Operations

Original Standards	Revised Standards
<p>Apply and extend previous understandings of arithmetic to algebraic expressions.</p> <p>6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as $5 - y$.</i></p> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</i></p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.</i></p>	<p>Apply and extend previous understandings of arithmetic to algebraic expressions.</p> <p>6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as $5 - y$.</i></p> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</i></p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.</i></p>



Geometry Progression



Polygons and Properties

Original Standards	Revised Standards
<p>Reason with shapes and their attributes. 3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p>	<p>Reason with shapes and their attributes. 3.G.1 Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).</p>
<p>Classify two-dimensional figures into categories based on their properties. 5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i></p>	<p>Classify two-dimensional figures into categories based on their properties. 5.G.3 Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).</p>
<p>Classify two-dimensional figures into categories based on their properties. 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.</p>	<p>Classify two-dimensional figures into categories based on their properties. 5.G.4 Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.</p>



Polygons and Properties (cont.)

Original Standards	Revised Standards
<p>Draw, construct, and describe geometrical figures and describe the relationships between them.</p> <p>7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p>	<p>Draw, construct, and describe geometrical figures and describe the relationships between them.</p> <p>7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions.</p> <p>a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p> <p>b. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.</p>
<p>New Standard</p>	<p>Classify and analyze geometric figures.</p> <p>G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.</p>



Symmetry

Original Standards	Revised Standards
<p>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</p> <p>4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>	<p>Deleted Standard</p>
<p>Experiment with transformations in the plane.</p> <p>G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.</p>	<p>Experiment with transformations in the plane.</p> <p>G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.</p> <ul style="list-style-type: none">a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.



Key Features and Challenges

- Checks and balances
- Getting the right people involved
- Higher education involvement
- Encouraging learning through the standards



Continuing Needs

- More focus in the high school
- More polish in the geometry progression
- Support for proof, in more than geometry
- Support for career-intending students